Extending the application of the relativity principle: Some pedogogical advantages

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(Received 7 December 1995; accepted 4 June 1996)

Although textbooks usually present the conservation laws of energy and momentum only in the laboratory frame of reference, it is important that an introductory physics course show that they are valid in any inertial frame. It can be useful to apply the conservation laws in more than one frame of reference so as to highlight that they are precise only in closed systems. This is of particular note if one of the interacting partners is of near-infinite mass, whose share of the redistributed energy cannot always be neglected. In this sense, the notion of ''infinitely large mass'' is frame dependent. Balancing the energy and momentum in more than one frame of reference can help resolve some common difficulties with regard to energy conservation. We show also that the energy-work theorem holds in noninertial frames where inertial forces are treated as external.

I. INTRODUCTION

In most universities and colleges, standard Introductory Physics Courses (IPC) introduce ideas of multiple representations of physical reality in different frames of reference and the relationship between those representations. Multiple representations are related to the principle of invariance, and are important because, as Resnick, Halliday, and Krane claim, ''Invariance principles often give us a clue about the working of the natural world; they signal that a particular relationship is not an accident of one observer's preferred position but is instead an effect of some deep underlying symmetry of nature.''[1]

An IPC usually starts with kinematics, where the relative nature of motion is asserted and velocity transformation is established in its Galilean form:

$$\mathbf{v}=\mathbf{v}'+\mathbf{u}, \tag{1}$$

where $\mathbf{v}$ is the velocity of an object in the original frame of reference, and $\mathbf{v}'$ the velocity in another frame moving relative to the original with velocity $\mathbf{u}$. In this context the principle of relativity is usually stated for the first time in the course. This principle claims the equivalence of all observers in application of physical laws to describe natural phenomena—a very general statement that cannot be checked by students at the time it is introduced.

When Newton's laws are presented in the unit on dynamics, it would be natural to apply the relativity just learned and consider different frames of reference; few texts do so. Rarely is the concept of an inertial frame of reference even defined. A notable exception is Tipler's: ''A reference frame in which Newton's first law holds is called an inertial reference frame.''[2] Such statements as ''There is no single inertial frame of reference that is preferred over all others for formulating Newton's laws''[3] when made, are often left without justification although it does not need any sophisticated procedures to *show* that Newton's laws match this condition.[4] Relation (1) with constant $\mathbf{u}$ directly provides the independence of acceleration on the observer:

$$\mathbf{a}=\mathbf{a}'. \tag{2}$$

Using (2), and under the additional assumption of force invariance with change of inertial frame of reference, one can infer that Newton's second law, $m\mathbf{a}=\mathbf{F}$, remains valid for all inertial observers.

In regard to momentum and energy, IPC texts do not usually consider more than one observer.[5] This is somewhat surprising as the expressions for energy and momentum are velocity dependent so it is a valid question to ask whether energy/momentum conservation laws satisfy the relativity principle. In two texts,[6,7] we found this problem addressed in an optional addendum, but only briefly.

The laws of momentum and energy conservation are of central importance; however, they should not be applied automatically. For them to hold at all requires that the system be closed, that is, its objects should be considered as interacting pairs. This constraint is not often appreciated and may be seen as too ''academic,'' or of little ''practical'' value. Students holding these views are on shaky ground facing the serious difficulties illustrated in Sec. III.

II. CONSERVATION LAWS AND THE PRINCIPLE OF RELATIVITY

The reason why Newton's second law ($\mathbf{F}\Delta t=\Delta\mathbf{p}$) remains valid under the Galilean transformation of velocities is because momentum is a linear function of velocity, and force is invariant in classical mechanics. The observer invariance of the conservation of linear momentum in a system of colliding particles can be shown directly. Suppose observer S^0 claims

$$\sum_j m_j\mathbf{v}_j^i=\sum_j m_j\mathbf{v}_j^f, \tag{3}$$

where $\mathbf{v}_j^i$ stands for the initial velocity of the jth particle and $\mathbf{v}_j^f$ the velocity of the same particle, following a physical interaction. Then the moving observer S infers:

$$\sum_j m_j(\mathbf{v}_j^i+\mathbf{u})=\sum_j m_j(\mathbf{v}_j^f+\mathbf{u}), \tag{4}$$

which is true under the constraint of the closed system. Such an invariance regarding energy conservation, however, is not obvious. The potential energy is velocity independent and so does not present a problem, but for the kinetic energy of the same system, observer S^0 claims (in the case of elastic collisions):

$$\sum_j \frac{1}{2}m_j(v_j^i)^2=\sum_j \frac{1}{2}m_j(v_j^f)^2. \tag{5}$$

Meanwhile, the same process is described by observer's S:

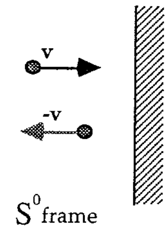
$$\sum_j \frac{1}{2}m_j(\mathbf{v}_j^i+\mathbf{u})^2=\sum_j \frac{1}{2}m_j(\mathbf{v}_j^f+\mathbf{u})^2. \tag{6}$$

Thus we obtain

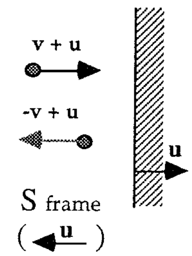
$$\sum_i\left(\frac{1}{2}m_j(v_j^i)^2+m_j\mathbf{v}_j^i\mathbf{u}+\frac{1}{2}m_ju^2\right)=\sum_i\left(\frac{1}{2}m_j(v_j^f)^2+m_j\mathbf{v}_j^f\mathbf{u}+\frac{1}{2}m_ju^2\right),$$

which only holds true because momentum is conserved (3). It is therefore essential that the system is closed. This fact is worthy of attention.

Moreover, the objects in this example were treated as point masses of undefined magnitude and no work terms explicitly appeared in the energy balance. In more realistic situations, difficulties start to appear when the relativity principle is applied to balance work and energy, especially where deformable bodies are treated in open systems[8] or one of the objects is extremely massive.



(a)



(b)

Fig. 1. One-dimensional elastic collision of a ball with a solid wall in the wall's rest-frame S^0 (a), and in frame S, moving with velocity relative to the wall (b).

III. REPRESENTATIVE EXAMPLES AND DISCUSSION

(1) Consider the simplest example of an interaction involving an object with near-infinite mass: one-dimensional elastic collision of a ball against a solid wall. This common example is usually considered in the wall's rest-frame, S^0 [Fig. 1(a)]. Note that the momentum of the ball is not conserved, but its energy seemingly is (the collision is elastic). The lack of symmetry between energy and momentum may puzzle the novice learner. This is because the ball does not comprise a closed system so, in fact, neither the momentum nor energy of the ball are conserved. Nevertheless, while one *can* neglect the energy transferred to the wall (elastic collision), one *cannot* neglect the transferred momentum. As a consequence, some learners perceive the idea of energy conservation of the *ball*.

In fact, as regards the wall–ball system, one can say that momentum was redistributed by the collision, but the energy was not. It remained with the ball. Why? Here the infinite mass played its role. In S^0 one may account for the energy and momentum in the collision:

$$\tfrac{1}{2}(mv_b^i)^2=\tfrac{1}{2}(mv_b^f)^2+\tfrac{1}{2}(Mv_w^f)^2 \tag{7}$$

and

$$mv_b^i=mv_b^f+Mv_w^f, \tag{8}$$

where i indicates initial and f the final velocities of the ball and wall of masses m and M, respectively (v_b and v_w). After solving for the final velocities, one obtains

$$v_b^f=-v_b^i\,\frac{M-m}{M+m}\quad\text{(for the ball)}, \tag{9}$$

$$v_w^f = 2v_b^i \frac{m}{M+m} \quad \text{(for the wall).} \tag{10}$$

As the wall is infinitely larger than the ball ($m \ll M$),

$$v_b^f = -v_b^i, \quad v_w^f = 0. \tag{11}$$

Consider now another frame S, moving with velocity $\mathbf{u}$ relative to the wall [Fig. 1(b)]. In this frame, the change in momentum is the same as it was in S^0. However, the initial energy of the ball is now $\frac{1}{2}m(u+v)^2$ and its energy after the collision is $\frac{1}{2}m(u-v)^2$, they are no longer equal. This fact should only come as a surprise to the learner who insisted on conserving the energy of *the ball* in S^0 without a second thought. Thus, observer S, finds that velocities of the ball are $v_b^i + u$ before and $v_b^f + u$ after the collision. The corresponding velocities of the wall are u and $v_w^f + u$. So, the total mechanical energy of the wall–ball system is initially

$$E^i = \tfrac{1}{2}m(v_b^i + u)^2 + \tfrac{1}{2}Mu^2 \tag{12}$$

and, finally,

$$E^f = \tfrac{1}{2}m(u + v_b^f)^2 + \tfrac{1}{2}M(u + v_w^f)^2. \tag{13}$$

Because of (9) and (10), the energies (12) and (13) coincide thus ensuring that the energy is conserved in an arbitrary inertial frame.

Solving this simple example which, at a first glance, contradicts the equivalence of the observers, may stimulate students to make a conceptual analysis of the situation leading to their rediscovering of the importance of a *closed* system for energy–momentum conservation. In S^0 one can neglect the wall in the energy balance either intentionally or through lack of awareness. In frame S, the wall cannot be neglected; the contributions of *both* the ball and the wall are equally important to balance both momentum and mechanical energy. In S^0 the velocity gained by the wall during impact is small but its momentum Mv_w^f must be of the same order of magnitude as that of the ball, mv_b^f. Obviously, the equality of magnitudes established within the linear dependence does not hold in the product $M(v_w^f)^2$ which can, therefore, be neglected. From an arbitrary frame S, the energy balance is different due to the initial energy and momentum of the wall.

(2) The treatment of an explosion (an inelastic collision reversed in time) introduces internal, nonmechanical energy into the discussion. Suppose a cannon, *fixed* to the ground, shoots a ball with speed v. An observer on the ground, S^0, accounts for the energy balance:

$$\frac{mv^2}{2} = \Delta E_{\text{int}}, \tag{14a}$$

where ΔE_{int} is the change of the internal energy of the explosives converted into kinetic energy of the ball m. The cannon's share of the kinetic energy is neglected in (14a), apparently on the grounds of the near-infinite mass of the cannon. It is not crucial for the observer to be aware of the interaction between the cannon and the ball. This is not the case for an arbitrary observer S, moving at velocity $\mathbf{u}$ (Fig. 2). By neglecting the cannon, observer S, would be making an error in the energy balance:

$$\frac{(m+M)u^2}{2} + \Delta E_{\text{int}} = \frac{m(v+u)^2}{2} + \frac{Mu^2}{2}. \tag{15a}$$

Equations (14a) and (15a) cannot be satisfied by the same amount of internal energy ΔE_{int}. This already presents a

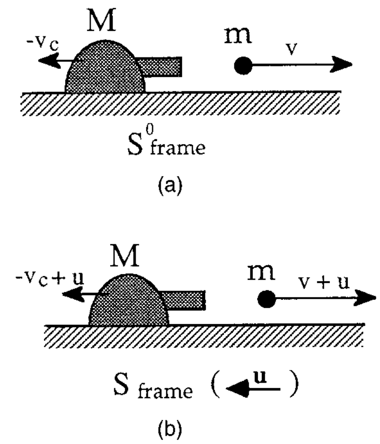


Fig. 2. A cannon fixed on the ground, shoots a ball with a speed v as it is viewed in the ground's frame S^0 (a), and in the frame S, moving relative to the ground (b).

paradox: the change of the internal energy must be invariant,[9] or, in other words, the amount of explosives used cannot be greater for an observer driving past the cannon. The paradox is resolved when we discard the tacit assumption that the change in the cannon's kinetic energy is negligible. Indeed, taking into account the reaction velocity of the cannon, v_c, as recorded in S^0 [Fig. 2(a)], one rewrites (14a):

$$\frac{Mv_c^2}{2} + \frac{mv^2}{2} = \Delta E_{\text{int}}. \tag{14b}$$

To observer S [Fig. 2(b)], the same energetic balance is given by:

$$\frac{(M+m)u^2}{2} + \Delta E_{\text{int}} = \frac{M(u-v_c)^2}{2} + \frac{m(v+u)^2}{2}. \tag{15b}$$

Thus, the views of S^0 and S, Eqs. (14b) and (15b), become consistent; they are satisfied by the same ΔE_{int}. In checking this fact one needs to use the momentum conservation $mv - Mv_c = 0$.

These first two examples illustrate the same idea. Neglect of the near-infinite mass in the energy balance may misrepresent the interaction and mislead the learner. In general, interaction will change the kinetic energy of *all* interacting partners, no matter how big they are. Only by taking this into account does the energy conservation become observer-invariant. Comparing the descriptions in S^0 (where the massive component is at rest) and in another frame S highlights this point.

(3) We now consider interactions spatially and temporally extended. The work done during an interaction must now be included in balancing the energy. Textbooks treat such processes exclusively in a laboratory frame of reference though any inertial frame would be equally valid according to Galileo's principle.

Suppose a ball, initially at height h, slides down along a perfectly smooth curved track [Fig. 3(a)]. The track is fixed to the ground. Students are often asked to predict the final velocity of the ball. Without difficulty, most equate the potential energy of the ball (often mistakenly understood to be possessed by the ball itself) to its kinetic energy at the base:

$$mgh = \tfrac{1}{2}mv^2. \tag{16}$$

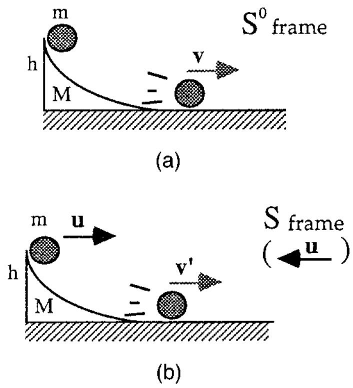


Fig. 3. A ball slides down along a perfectly smooth curved track which is fixed to the ground. The final velocity of the ground is neglected: (a) the view in the ground's frame S^0; (b) the view in the frame S, moving relative to the ground.

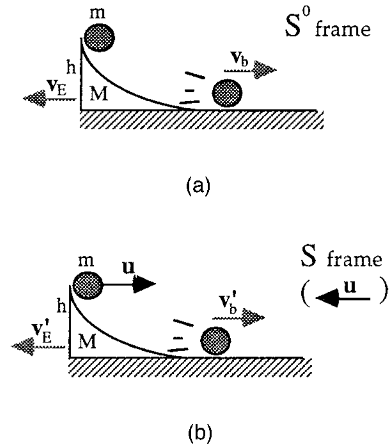


Fig. 4. A ball slides down along a smooth track fixed to the ground. The final velocity of the ground is not neglected: (a) the view in the frame S^0, in which the ball was initially at rest; (b) the view in the frame S, moving relative to the ground.

This implies for the final velocity

$$v=\sqrt{2gh}. \tag{17}$$

But, suppose one considers the same event in the moving frame S [Fig. 3(b)]. Observer S perceives the initial speed of the ball u, and, by the application of (1) to (17), the final speed v' of the ball: $v'=\sqrt{2gh}+u$. There the initial energy is given by

$$E_i=mgh+\tfrac{1}{2}mu^2, \tag{18}$$

whereas the final energy is

$$E_f=\tfrac{1}{2}m(\sqrt{2gh}+u)^2. \tag{19}$$

Hence, $E^i=E^f$ is only true for $u=0$. This would imply that the rest frame is unique, in perfect accord with Aristotle's assertion.

The incompatibility of Eqs. (18) and (19) arises from neglecting the fact that the ball alone does not present a closed physical system. But, due to the near-infinite mass of the earth, neglecting it in S^0 does not cause a significant numerical mistake. But is this so in other frames?

The correct treatment for energy conservation in S^0 [Fig. 4(a)] is

$$mgh=\tfrac{1}{2}m(v_b^f)^2+\tfrac{1}{2}M(v_E^f)^2, \tag{20}$$

where m and M are the masses of the ball and the Earth, and v_b^f and v_E^f their final speeds.[10] Conservation of the linear momentum provides

$$mv_b^f+Mv_E^f=0. \tag{21}$$

Solving Eqs. (20) and (21), one obtains

$$v_b^f=\sqrt{\frac{2gh}{\left(1+\frac{m}{M}\right)}}, \tag{22}$$

$$v_E^f=-\frac{m}{M}\sqrt{\frac{2gh}{\left(1+\frac{m}{M}\right)}}, \tag{23}$$

and, for the final relative speed v_r of the ball,

$$v_r=v_b^f-v_E^f=\sqrt{2gh\left(1+\frac{m}{M}\right)}. \tag{24}$$

Obviously, if $M\gg m$, $v_r\approx v$, and the result (17) is confirmed but only as an approximation.

The same process in the moving frame S [Fig. 4(b)], has the ball with an initial speed u. The final speeds are $v_b'^f$ for the ball and $v_E'^f$ for the Earth. Balancing the energy leads to

$$mgh+\tfrac{1}{2}mu^2+\tfrac{1}{2}Mu^2=\tfrac{1}{2}m(v_b'^f)^2+\tfrac{1}{2}M(v_E'^f)^2 \tag{25}$$

which, together with the momentum conservation,

$$mu+Mu=mv_b'^f+Mv_E'^f, \tag{26}$$

yields the solution

$$v_b'^f=u+\sqrt{\frac{2gh}{\left(1+\frac{m}{M}\right)}} \tag{27}$$

and

$$v_E'^f=u-\frac{m}{M}\sqrt{\frac{2gh}{\left(1+\frac{m}{M}\right)}}. \tag{28}$$

Thus the speed v_r of the ball relative to the Earth is still given by

$$v_r'=v_b'^f-v_E'^f=\sqrt{2gh\left(1+\frac{m}{M}\right)} \tag{29}$$

as it was in (24). This confirms the validity of energy conservation in any arbitrary inertial frame S under the condition that the system be closed. Only when it is, does Galileo's principle hold.

This example also reveals the *interactive* nature of forces. The elastic force acts *equally* on the ball and on the track, changing kinetic energies of both. By using an arbitrary frame of reference this fact can be made more explicit in a way which might be unique in a classroom.

The fact that changes in the energy of the Earth must be taken into account might seem odd: in frame S^0 we neglected them on the grounds of Earth's ''infinite mass.'' Surely, the fact that the Earth is near-infinitely bigger than any other object, seems to be observer-invariant. However, as we have seen, in an arbitrary frame of reference S, the Earth's share in the energy balance may be of the same magnitude as those of other objects and cannot be neglected, as it was in S^0. If this act of neglecting was related to the fact of the near-infinite mass of the Earth, one could now consider the meaning of the notion ''infinite mass'' as frame dependent.

(4) The energy-work theorem equates the change in kinetic energy of a system to the net work done by all active forces: $(\Delta E_{\text{kin}} = \Sigma_i W_{F_i})$.[11] Though its application is not restricted to closed systems (in the case of a closed system, this theorem can be reduced to energy conservation), difficulties may arise when accounting for some everyday experiences (jumping, climbing, accelerating car) which cannot be described on the *macroscopic* level in terms of work as it is normally defined.[12] In an attempt to solve this difficulty a new concept of ''pseudowork'' was introduced, which is appropriate to describe *open* systems and deformable objects.[13] Sherwood[14] clarified the existing dichotomy in energy-work macroscopic descriptions of physical systems, distinguishing between the energy-work theorem (or, beyond mechanics, the first law of thermodynamics), and the C.M. (center of mass) equation. Which of the two descriptions should apply depends on the specific interests of the user. Penchina[13] suggests that pseudowork could provide simpler solutions in specific cases, but the standard textbooks have not yet adopted this concept. Our study focuses on energy-work descriptions of *closed* systems and suggests an alternative physical interpretation.

Suppose a boy is standing on a large platform moving to the right with a speed $v/2$ [Fig. 5(a)]. He throws a stone, m, to the left with a relative speed v [Fig. 5(b)]. Relative to the Earth [frame S, Fig. 5(c)], the stone is then observed moving at a speed $(-v/2)$. The energy-work theorem applied by the ground observer S may provide an odd result. The stone retains its kinetic energy, so if, for any reason, the kinetic energy of the massive platform was neglected, a paradox emerges: was there any work performed? The act of throwing is obviously energy consuming, then how can one account for it? In fact, in this interesting case, the effort applied by the boy is entirely invested in the change of the kinetic energy of the ''infinitely massive'' platform (which has been accelerated).

Let us first take the view of the observer S^0 who is located on the platform. The act of throwing is perceived as extended along the distance d_0, and time, t^*. The energy balance is described as follows:

$$\Delta E_{\text{stone}}^{\text{kin}} = W_{\text{on stone}}^{S_0} = \Delta E_{\text{boy}}^{\text{int}}, \tag{30}$$

where

$$\Delta E_{\text{stone}}^{\text{kin}} = \frac{mv^2}{2}, \quad W_{\text{on stone}}^{S_0} = Fd_0. \tag{31}$$

Therefore

$$\frac{mv^2}{2} = Fd_0 = \Delta E_{\text{boy}}^{\text{int}}. \tag{32}$$

F is the force applied by the boy on the stone, and $\Delta E_{\text{boy}}^{\text{int}}$ is the internal energy invested by the boy. In S^0, friction forces between the platform and the boy do not perform any work on a stationary platform (they perform pseudowork[13,14]).

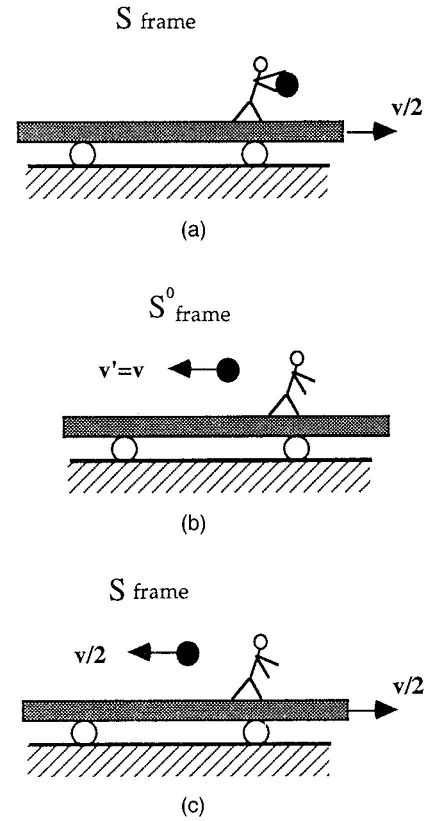


Fig. 5. (a) A boy is standing on a massive platform, moving to the right relative to the ground (view in frame S fixed to the ground). The boy throws a stone to the left with a speed v relative to himself as viewed in the frame S^0, of the platform (b), and as is viewed in the frame S (c).

Another observer, on the ground frame S, accounts for the same process differently. The work of the force on the stone is $W_{\text{on stone}}^{S} = Fd$, but the displacement d along which the force was applied on the stone is (in S): $d = d_0 - (v/2)t^*$. As the time of throwing can be estimated as $t^* = d_0/(v/2)$ one obtains $d=0$ (the stone approximately remains in the same location when being thrown), which implies zero work on the stone. The only force that performs a work in S is the friction force applied on the platform. This work is: $W_{\text{on platf}}^{S} = Fd_{\text{platf}}$. An estimation of the platform displacement during the throwing yields: $d_{\text{platf}} = (v/2)t^* = d_0$. Then, $W_{\text{on platf}}^{S} = Fd_{\text{platf}} = Fd_0$ exactly reproduces the work on the stone in S_0 (31). Therefore, in the frame S the energy-work balance becomes

$$\Delta E_{\text{platf}}^{\text{kin}} = W_{\text{on platf}}^{S} = \frac{mv^2}{2} = \Delta E_{\text{boy}}^{\text{int}}. \tag{33}$$

This equation reflects the same loss of chemical energy $\Delta E_{\text{boy}}^{\text{int}}$ within the boy. Summarizing we see that as far as the closed system is treated, energy conservation (the energy-work theorem) is equally valid for both observers' descriptions. The difference between the observers' descriptions lies in their identification of works done by the active forces. As far as a closed system is considered, the interpretation of the energy-work balance does not require the pseudowork concept.

In fact, this same example may be even more impressive without involving a platform. A walking boy throws a stone

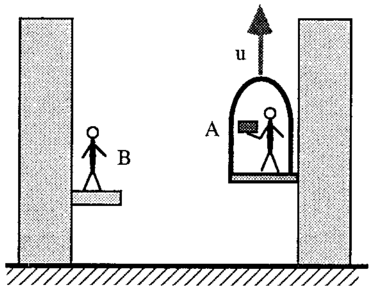


Fig. 6. A person is holding an object in an ascending elevator. The situation is viewed in two frames: A—of the elevator, and B—of the Earth. In A, the passenger does not do any work. From the point of view of B, the passenger does work.

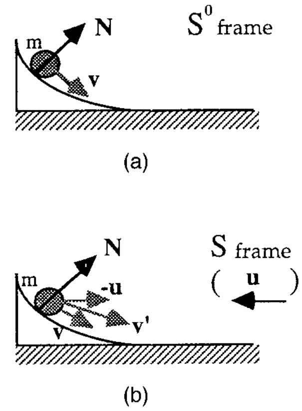


Fig. 7. In the ground's frame S^0, the contact elastic force acts on the ball at right angles to the surface and does not perform work (a). In frame S, moving to the left, the contact force is not perpendicular to the ball's velocity and does perform work (b).

backwards. The unchanged kinetic energy of the stone, as perceived by an on ground observer at rest, implies that the second partner in the interaction, namely the Earth, is important in the redistribution of energy. The Earth is accelerated by an infinitely small amount, but absorbs a finite amount of energy and momentum. It is this which ensures that the energy and momentum balance. Although this inference may be surprising, the other main assertion should no longer be so: to account correctly for the energy and momentum one may find it necessary to consider a closed physical system.

(5) The energy-work theorem is unlike the law of energy conservation in that it may be used to describe open physical systems. Then the work done by external forces may appear explicitly. Any change in the kinetic energy of objects is obviously frame dependent. Therefore, it is important to show that the work done by external forces has a similar frame dependence, such that the validity of the energy-work theorem is preserved.

Halliday *et al.*[16] consider the example of a person holding an object in an ascending elevator (Fig. 6). The situation is treated in two frames: A— of the elevator, and B— of the Earth. In A, the passenger does not do any work, as the object he holds is at rest. The kinetic energy does not change (neglecting microscopic trembling of the passenger's hand). From the point of view of B, the situation is different and work is done by the passenger. This example, however, might not seem sufficiently general as the net work (of gravitational force and contact force) is zero for both observers. Although Resnick *et al.* considered a more general example[17] to infer the validity of the energy-work theorem in an arbitrary frame, they did not explain the specific role of the ''very massive'' partner in the interaction (the Earth or platform). To show its role one can consider the same energy-work balance in more than one frame of reference, as we did in our examples.

It is instructive to expose the ''mechanism'' by which changes in the amount of kinetic energy and in the values of work reflect the change of the frame. For example, while in frame S^0 [Fig. 7(a)], the contact elastic force acts on the ball at right angles to the surface, hence, perpendicular to the ball velocity, it is no longer so in frame S [Fig. 7(b)], where velocity $\mathbf{u}$ is added to each velocity. Therefore, the contact force does perform work in S. This work causes the greater increase of kinetic energy as measured in the S frame. Most authors, trying to simplify the presentation, do not specify the frame of reference in which they consider energy transfer and do not define the physical system; this makes it impossible to differentiate between internal and external forces.

(6) Is the energy-work theorem valid in noninertial frames of references? The answer is not obvious.[18] The description of reality in such frames involves inertial forces. These forces are commonly left out of IPC textbooks and standard high school curricula in spite of the natural tendency of novice learners to interpret their sensory experience in terms of such forces.[19] Moreover, such ideas are ordinarily interpreted by the instructor as a force–motion misconception. However, it is legitimate to describe physical processes in noninertial frames. Newton's laws and energy-work theorem hold in noninertial frames with the addition of inertial forces. Inertial forces are not interactive and do not appear in action–reaction pairs. Consequently, inertial forces should be always treated as external, and this is the only difference between them and real forces.

Let us consider an alternative account for example (3) in the frame of the ball itself, S^b, which presents a noninertial frame. Observer S^b, attached to the ball, perceives the track and the Earth (not the ball) moving [Fig. 8(a)]. The kinetic energy of the ball remains zero, and the track (Earth) accelerates from rest to its final velocity, equal in magnitude and opposite in direction of v_r, (24). The total mechanical energy of the ball–Earth system in S^b is

$$E^i = mgh \quad \text{(initially)}, \quad E^f = \tfrac{1}{2} M v_r^2 \quad \text{(finally)}. \tag{34}$$

By direct use of Eqs. (20), (22), and (23) for mgh, and Eq. (24) for v_r, one obtains the total change of the mechanical energy:

$$\Delta E = E^f - E^i = Mgh. \tag{35}$$

This result could have been foreseen qualitatively. The work W_{in} done by the inertial force F_{in} can be calculated along the curved trajectory of the track. Each infinitesimal piece of this trajectory could be considered as rectilinear, and inclined at angle α varying from point to point. Since the acceleration of the sliding ball in the frame of the Earth, S^0, is well known, $g \sin \alpha$, the acceleration of the track (Earth) in the frame of the ball, S^b, can be easily ascertained. The relations $d\mathbf{r}_{\text{track}} = -d\mathbf{r}_{\text{ball}}$ and $\mathbf{a}_{\text{track}} = -\mathbf{a}_{\text{ball}}$ between the dis-

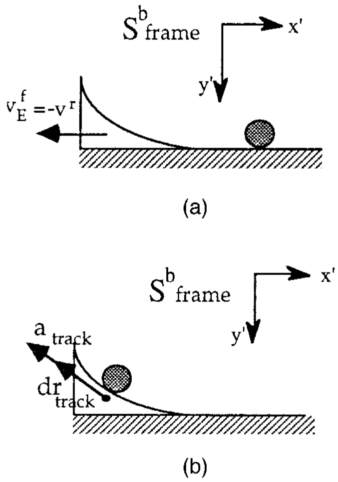


Fig. 8. (a) Example (3) as viewed in the noninertial frame S^b of the ball itself. (b) Acceleration and displacement of the track as viewed in frame S^b.

placements and accelerations in the two frames are obvious [Fig. 8(b)]. Therefore, the elementary work performed by the inertial force is

$$dW_{\mathrm{in}.f}=\mathbf{F}_{\mathrm{in}}\cdot d\mathbf{r}_{\mathrm{track}}=(-M\mathbf{a}_{\mathrm{ball}})\cdot(-d\mathbf{r}_{\mathrm{ball}})$$
$$=M\mathbf{g}\cdot d\mathbf{r}_{\mathrm{ball}}=Mgdh, \quad (36)$$

where $\mathbf{r}_{\mathrm{track}}$ and $\mathbf{a}_{\mathrm{track}}$ are measured in S^b, $\mathbf{a}_{\mathrm{ball}}$ and $\mathbf{r}_{\mathrm{ball}}$—in S^0. dh is a vertical component of the displacement $d\mathbf{r}_{\mathrm{ball}}$. We have used here an evident fact that in S^0 $\mathbf{a}_{\mathrm{ball}}\cdot d\mathbf{r}_{\mathrm{ball}}=\mathbf{g}\cdot d\mathbf{r}_{\mathrm{ball}}$. Finally, integrating over the whole drop, one obtains the total work done by the inertial force:

$$W_{\mathrm{in}.f}=W_{\mathrm{tot}}=Mgh, \quad (37)$$

The results (37) and (35) exactly coincide, confirming the validity of the energy-work theorem ($\Delta E = W_{\mathrm{in}.f}$) in frame S^b. For the observer S^b, the change in mechanical energy is the change of the Earth's energy (numerically enormous), and is due to the work performed by the inertial force. Accounting for inertial forces in the energy-work balance appears to be the same as accounting for any interactive external forces. This conclusion could be foreseen on the basis of Einstein's equivalence principle.

(7) Is there an inertial frame which would allow us to neglect the contribution of the Earth, thus greatly simplifying the energy description of physical processes? To answer this question, we compare the changes in kinetic energies [of the ball and the Earth in example (3)] as measured in an arbitrary inertial frame S. Using Eqs. (27) and (28), one obtains

$$\Delta E_{\mathrm{ball}}=\tfrac{1}{2}\,(m(v_b'^{f})^2-mu^2)$$
$$=\frac{mgh}{1+\frac{m}{M}}+\frac{mu\sqrt{2gh}}{\sqrt{1+\frac{m}{M}}}\approx mgh+mu\sqrt{2gh}, \quad (38)$$

$$\Delta E_{\mathrm{Earth}}=\tfrac{1}{2}\,(M(v'^{f}_{E})^2-Mu^2)$$
$$=-\frac{mu\sqrt{2gh}}{\sqrt{1+\frac{m}{M}}}+mgh\,\frac{\left(\frac{m}{M}\right)}{1+\frac{m}{M}}\approx-mu\sqrt{2gh}. \quad (39)$$

This means that under the condition

$$mu\sqrt{2gh}\ll mgh \quad (40)$$

or

$$u_{\text{(frame velocity relative to the earth)}}\ll v_{\text{(final object velocity in } S^0)}, \quad (41)$$

one can neglect the contribution of the earth's kinetic energy.

The terms which are dependent on the object-to-earth mass ratio were finally neglected in (38) and (39).[20] Conditions (40) or (41) explain why it is preferable to describe the energy of a physical process from the Earth's rest frame. This point, once grasped, may enhance the correct understanding of conservation laws with the awareness of redistribution of energy following the force interaction between different objects. Thus it explains the most quoted formula in introductory physics texts, $\frac{1}{2}mv^2=mgh$, which only applies because we do not live on a small asteroid, where this formula would be a mistake. If we did live on such an asteroid, the problem-solving in physics classes would be much more complicated.

IV. CONCLUSION

This paper has made three suggestions. First, that undergraduate students could better understand the problem of measuring energy-momentum balances when they are treated in more than one frame of reference. Seeing the laws of energy-momentum conservation (or the energy-work theorem) in different frames of reference, changing the viewpoint, requires us to treat all interaction partners and makes explicit the closed-system constraint. Second, we have shown how the neglect of ''infinitely large masses'' may lead to mistakes, and also that use of multiple frames can help to solve this problem. Third, we have suggested an application of the energy-work theorem to account for energy-work balances in noninertial frames of reference where inertial forces should be treated as external. This is in accord with Einstein's principle of equivalence.

In general, viewing the process from the perspective of the relativity principle is conceptually beneficial as it helps students operationally assimilate the laws of conservation, the mutual nature of force interaction, and Galileo's relativity principle.

ACKNOWLEDGMENT

We thank an anonymous AJP reviewer who made stimulating comments on the early version of this paper.

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[3]H. D. Young, *Physics* (Addison-Wesley, Reading, MA, 1992).
[4]R. Resnick, D. Halliday, and K. Krane, in Ref. 1, p. 65.
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[6]D. Halliday, R. Resnick, and J. Walker, *Fundamentals of Physics* (Wiley, New York, 1993).
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[10]On first glance the expression for potential energy is strongly asymmetrical. The equivalent expression for this term is $E_{\mathrm{grav}}^{\mathrm{pot}} = -GMm/(R_E + h)$. It is not dependent on the choice of inertial frame of reference and can be replaced with good accuracy by the famous ''mgh'' term (plus a constant). The acceleration g accounts for the contribution of the Earth's mass.
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[14]B. A. Sherwood, ''Pseudowork and real work,'' Am. J. Phys. **51**, 597–602 (1983).
[15]In S^0 for the uniformly accelerated stone the average speed during the acceleration time t^* is $v/2$. t^* is, of course, frame independent. So the time t^* can be estimated by $t^* = d_0/(v/2)$.
[16]D. Halliday, R. Resnick, and J. Walker, in Ref. 6, p. 177.
[17]R. Resnick, D. Halliday, and K. Krane, in Ref. 1, p. 141.
[18]As stated in R. Resnick, D. Halliday, and K. Krane, Ref. 1, ''the laws of classical mechanics are valid only in a certain set of reference frames'' (p. 79).
[19]For example, I. Galili and V. Bar, ''Motion implies force: Where to expect vestiges of the misconception?'' Int. J. Sci. Ed. **14** (1), 63–81 (1992).
[20]Our reviewer suggested that the kinetic energy of the Earth could be neglected in any frame of reference moving at right angles to the final velocities, v_f, although, for brevity, we have not included our confirmation of this claim. This confirmation is straightforward.