Energy transfer in electrical circuits: A qualitative account

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We demonstrate that the use of the Poynting vector for a model of the surface charge of a current carrying conductor can help qualitatively explain the transfer of energy in a dc closed circuit. The application of the surface charge model to a simple circuit shows that electromagnetic energy flows from both terminals of the battery, mainly in the vicinity of the wires (and not inside them) to the load where it enters and is converted into heat at a rate obtained from Ohm's law. © *2005 American Association of Physics Teachers.*
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I. INTRODUCTION

The explanation of physical phenomena is one of the major goals of physics. Aristotle noted that to explain phenomena four types of "causes" should be provided: material, formal, effective, and teleological.[1] Among these four, the second and third are especially important in physics education. The formal cause explains a phenomenon by its "form" reflected in a description, an empirical rule ("experiment shows") or a mathematical feature ("due to the decreasing denominator"). The effective cause suggests reasoning by a mechanism, often presuming a microscopic model, rather than a macroscopic description. For example, to explain the ideal gas law and related concepts, we need the particle model from statistical mechanics beyond the empirical gas laws of thermodynamics.

Simple electrical phenomena present a challenge for providing effective explanations that involve microscopic mechanisms. The reason is that for electricity and magnetism students can only indirectly confirm the theoretical interpretation of the microscopic processes taking place. Only macroscopic manifestations of the latter are a subject of measurements. In such a situation, formal and casual explanations are intrinsically interwoven and hardly distinguishable. Microscopic explanation, basing on a model and universal physical concepts, becomes especially important pedagogically for the meaningful learning of physics.

In this paper we draw attention to the fact that the explanations of the propagation of energy from the generator (battery) to the load in a simple dc circuit are superficial in most introductory textbooks.[2] A more meaningful account of the energy transport from the generator to the load is required.

II. THE PROBLEM

How can we account for the process of energy transformation from the electric potential energy of electrons in the battery to the heating of the resistor by the current in the circuit? An answer is not found in common textbooks even for the simplest circuit including a battery, connecting wires and a load. Statements, such as "If a charge element dq moves through the box from terminal a to terminal b, its potential energy will be reduced by dqV_{ab}, where V_{ab} represents the potential difference,"[3] valid for a single particle in a potential field, are applied to an electric circuit (a complex system of particles) basing solely on a general claim. "The conservation of energy principle tells us that this energy must appear elsewhere in some form or other."[3] This account obviously does not explain much about the circuit. Indeed, in the Feynman lectures we read:[4]

> "We ask what happens in a piece of resistance wire when it is carrying a current. Since the wire has resistance, there is an electric field along it, driving the current. Because there is a potential drop along the wire, there is also an electric field just outside the wire, *parallel to the surface* (Fig. 27-5). There is, in addition, a magnetic field which goes around the wire because of the current. The **E** and **B** are at right angles; therefore there is a Poynting vector directed radially inward, as shown in the figure. There is a *flow of energy into the wire all around*. It is of course, equal to the energy being lost in the wire in the form of heat. So our "*crazy*" theory says that the electrons are getting their energy to generate heat because of the energy flowing into the wire from the field outside. Intuition would seem to tell us that the electrons get their energy from being pushed along the wire, so the energy should be flowing down (or up) along the wire. But the theory says that the electrons are really being pushed by an electric field, which has come from some charges *very far away*, and that the electrons get their energy for generating heat from these fields. The energy *somehow* flows from the distant charges into a wide area of space and then inward to the wire." (emphasis added).

Feynman applied the Poynting vector (Fig. 1), which determines the rate of flow of the electromagnetic energy density, to show the direction of its propagation. The Poynting vector usually is introduced later in the introductory course in the context of electromagnetic waves and not applied later to electric circuits. However, the result of such an application and the resulting energy transfer in the circuit apparently did not satisfy Feynman. He wrote: "this theory is obviously nuts, somehow energy flows from the battery to infinity and then back into the load, is really strange."[4] Feynman, however, did not persist and left the problem for others to find a reasonable explanation. Can we say more about energy transfer in this simple circuit?

III. SOLUTION: THE SURFACE CHARGE

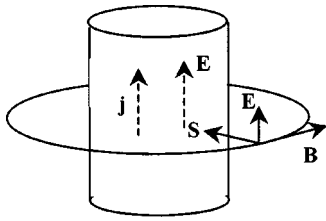The problem can be solved with a more adequate model for the electric current, which was available since the 1960s.

Fig. 1. The Poynting vector near a wire carrying a current (from Fig. 27-5 in Ref. 4).

Fig. 2. The rotation of the electric field $\mathbf{E}$ (E_n, E_τ) outside and along the closed dc circuit. Inside the wire the electrical field is collinear with the wire (E_τ). The case of a homogeneous wire and an ideal source with no internal resistance is considered.

It is simple to understand that a steady current in a wire implies a surface charge on the wire surface, guiding and pushing electrons. Sommerfeld[5] solved the problem of an infinitely long straight wire with a stationary current, with a return path through a coaxial cylinder surrounding the wire. The solution shows an electric field within and along the wire, E_τ, and a two-component field (axial E_τ and radial E_n) between the wire and the cylinder. This field configuration implies a jump of E_n on the wire surface, indicating a source, the surface charge.[6] Jefimenko[7] experimentally confirmed and visualized the surface charges of current-carrying conducting wires that produce an electric field. This result for a charged wire was surprising because most instructors presume that current carrying wires are electrically neutral (locally), or at least do not mention this fact. Model calculations of the surface charge for an infinite wire and for conductors of other geometries carrying direct current as well as RC circuits have been done.[8]

In physics education, the issue of the surface charge was first raised by Härtel.[9] A qualitative consideration of the surface charge was given by Chabay and Sherwood in their innovative text.[10] They emphasized the necessity of presenting microscopic models in physics instruction. However, the dissemination of the surface charge approach in learning materials is slow. Reference 10 remains in conceptual dissonance with most teaching materials in current use, which do not go beyond the Drude model of electrical current, and keep silent (or are mistaken) about the conceptual questions regarding microscopic processes in electrical circuits. We speculate that the reason is that unlike the Drude model and Kirchhoff's laws, the model of surface charge does not provide simple quantitative problems to facilitate assessment.

IV. ROTATION OF E

We will discuss the energy transfer in a simple closed circuit and provide a qualitative account. We first address the behavior of an electric field along the circuit.

We consider the distribution of the surface charge in a simple circuit comprised of a homogeneous wire (ρ=const) of uniform cross section. The gradient in the density of the surface charge provides the axial electric field within the wire that guides the movement of the conduction electrons (see Fig. 2).[11] The steady electric current in the resistive wire clearly implies an electric field of constant magnitude E_- inside the wire and collinear with it. This field is due to the distribution of surface charge established during the transient process. (The density gradient of the surface charge is constant for the simple case considered by Sommerfeld of an infinite and homogeneous axial wire.)

As shown in Fig. 2, the vector $\mathbf{E}$ has two components next to the wire and outside of it: E_n perpendicular and outward
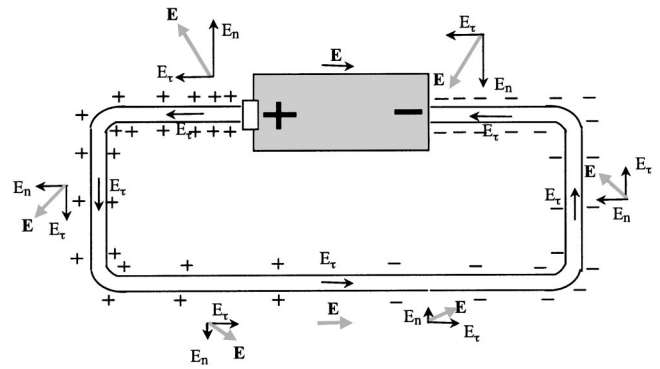
(or inward) to the wire, and E_τ along the wire, matching the sense of the current I (the direction of the current density $\mathbf{j}$). The normal component changes along the circuit in magnitude (reflecting the gradient of the surface charge) and in direction (reflecting the sense of the surface charge), while the tangential component remains the same magnitude within the wire and just next to it, because of the boundary conditions satisfied by the electric field.[12]

Although the direction of the electric field within the battery is clear (from plus to minus charge), there is a subtle point regarding the direction of the current there. The battery maintains a charge separation causing an electrostatic field between the terminals that resists this process. In fact, charge separation provides the *efficient* current[13] within the battery which closes the current loop in the entire circuit.

The Sommerfeld solution yields an electric field $\mathbf{E}$ that is perpendicular to the wire ($E_\tau = 0$) outside of it for an ideal conductor ($\rho \rightarrow 0$). The lack of a tangential field (no gradient of the surface charge on the wires[14]) is a reasonable result due to the inertial motion of electrons in the absence of resistance.[15] After adding a resistor to such a circuit, the surface charge will cause the electric field $\mathbf{E}$ to remain perpendicular to the ideal wires and gradually turn along the resistor (see Fig. 3).

In summary, $\mathbf{E}$ is directed from the positive to the negative terminal within the battery as well as inside the wires and the resistor. It is perpendicular to the wire surface outside the ideal wires connecting the battery to the resistor, and con-
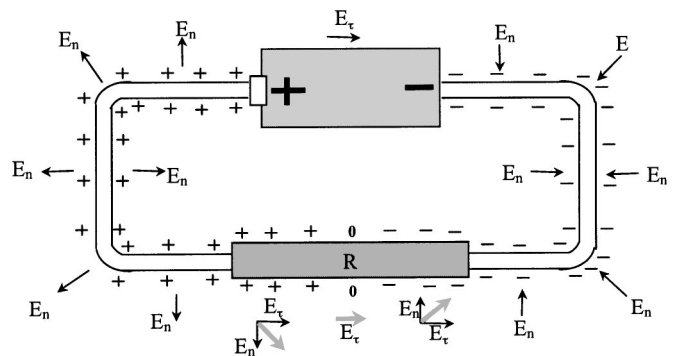

Fig. 3. The electric field $\mathbf{E}$ along and outside the closed dc circuit. The case of ideal wires, homogeneous resistor R, and ideal source with no internal resistance is considered.
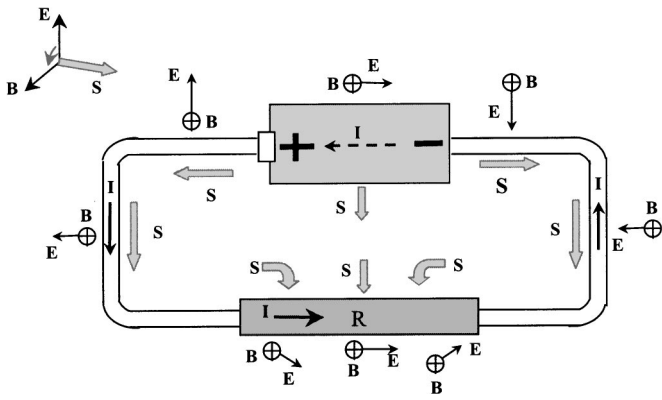
Fig. 4. The electric and magnetic fields, **E** and **B**, and Poynting vector **S** along the closed dc circuit. The case of ideal wires, homogeneous resistor R, and ideal source with no internal resistance is considered.

tinuously flips and changes in magnitude outside of the resistor (and real wires). Thus the **E**-vector reverses (by 180°) along the outer part of the circuit. This knowledge about **E**, as well as that of the current direction, enables us to determine the energy flux in the closed circuit.

V. THE POYNTING VECTOR IN THE CLOSED CIRCUIT

We now apply the Poynting vector, $\mathbf{S}=(1/\mu_0)$ (**E**×**B**), to describe the flow of the electrical energy.[16] The nature of the electric field in a simple circuit was described in Sec. IV, and the direction of the current yields the direction of the magnetic field. The magnetic field due to a linear current is familiar: force lines of concentric circles. Figure 4 shows **E** and **B** at different points along the circuit. The vector product shows the direction of the Poynting vector **S**. In Fig. 4 we show only the Poynting vector inside the loop and vectors **E** and **B** outside of it; **E** and **S** are axially symmetric (and **B** antisymmetric) to each local part of the circuit.

In the battery, the Poynting vector is outward, indicating the direction of energy flow. (Note the sensitivity of this result to the sense of the current through the battery.) In the vicinity of the conducting wires and next to the positive terminal of the battery, **S** is parallel to the wire. Perhaps surprisingly, **S** is directed *from* the battery on both sides of the battery. Along the resistor R, the change of direction of **E** outside the resistor causes **S** to change as well, gradually turning from parallel to perpendicular to the resistor axis (and entering it), at its middle point (zero surface charge).

Figure 4 demonstrates the rotation of the Poynting vector representing the energy flux from the generator, along the wires, eventually arriving at the resistor and delivering energy to it. The result is that electromagnetic energy flux is always directed from the source to the resistor and never returns (the resistor is heated).

This model also allows a simple evaluation of the energy flow rate. For an infinitely long wire, the magnetic field B equals $\mu_0 I/2\pi r_0$ on the surface of the cylindrical resistor with radius r_0. By using Ohm's law, $E=RI/L$, with L the length of the cylindrical resistor R, we obtain the flux of the Poynting vector through the surface A of the resistor:

$$ S = \frac{1}{\mu_0} \oint_A [E \times B]\mathrm{d}A = \frac{1}{\mu_0} EBA = I^2R. \tag{1} $$

This familiar result for the rate of transformation of the electromagnetic energy into heating in Ohmic resistor is covered earlier in a typical undergraduate course.

Although the model is qualitative, it allows an approximate evaluation of the decrease of the energy flux away from the wire. Because both the electric and magnetic fields decrease close to the current carrying wire as $1/r$, the Poynting vector decreases there as $1/r^2$.

VI. SUMMARY AND IMPLICATIONS FOR TEACHING

We have seen that even in the simplest dc circuit, the Poynting vector allows the visualization of the electromagnetic energy flux on its way from a source to a resistor. The significant aspects of this approach, which combines formal and causal explanations, include the following

(1) The Poynting vector conceptualizes and thus quantifies the transport of energy by the electromagnetic field. The Poynting vector is usually considered in undergraduate university physics courses[17] as a way of representing the energy flux of the electromagnetic wave. We have shown that it also can be useful for representing the energy flux in a closed dc circuit.

(2) The surface charge model is essential for students' understanding of energy transfer in the dc circuit. Past attempts to apply the Poynting vector for this purpose failed because of the neglect of the surface charge.

(3) Electromagnetic energy does not flow in the wires, as it might be intuitively assumed, but next to them. It enters into the resistors in the circuit at the rate of I^2R.

(4) Energy flow goes from both terminals of the dc battery to the load and never returns to the battery. Ironically, this understanding might look as if it supports the naïve "clashing currents model," a well-known misconception regarding the electric current in dc circuits.[18]

The following questions and tasks could be suggested to students: (1) Why should we expect the existence of the surface charge on a dc carrying wire without solving Maxwell equations? (2) Does the surface charge on the wires of the dc circuit violate the electroneutrality of the circuit? (3) How is the electric energy transferred in the dc circuit? (4) What is the role of energy dissipation in the dc circuit? (5) Why should we prefer the idea of electric energy transport next to the wires and not within them? (6) What is the physical reason that the electric field of the surface charge must be perpendicular to the wires in the case of zero resistivity wires? (7) Obtain the Poynting vector at the dc battery and explain the direction of the electric and magnetic fields in it. (8) Compare the energy dissipation rate in the resistor according to Ohm's law with the rate of flux of Poynting vector entering the resistor.

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[1]Aristotle, *Physics* (Peripatetic P. Grinnvel, IA, 1980), Vol. II, Chap. 3.

[2]We address instruction at the level of D. Halliday, R. Resnick, and J. Walker, *Fundamentals of Physics* (Wiley, New York, 2001). Most textbooks ignore the issue of energy transfer in dc electrical circuits.

[3]D. Halliday and R. Resnick, *Fundamentals of Physics* (Wiley, New York, 1988), p. 651.

[4]R. Feynman, R. Leighton, and M. Sands, *Feynman Lectures on Physics* (Addison–Wesley, Reading, MA, 1964), Vol. 2, pp. 27–28.

[5]A. Sommerfeld, *Electrodynamics* (Academic, New York, 1952), pp. 125–130.

[6]J. D. Jackson extended the axially symmetrical problem of Sommerfeld to the closed circuit of the same symmetry and provided a comprehensive and exhaustive analytic solution of this case. See J. D. Jackson, "Surface charges on circuit wires and resistors play three roles," Am. J. Phys. **64**(7), 855–870 (1996). See also J. A. Hernandes and A. K. T. Asis, "The potential, electric field and surface charges for a resistive long straight strip carrying a steady current," Am. J. Phys. **71**(9), 938–942 (2003).

[7]O. Jefimenko, "Demonstration of the electric fields of current-carrying conductors," Am. J. Phys. **30**, 19–21 (1962).

[8]See N. Preyer, "Surface charges and fields of simple circuits," Am. J. Phys. **68**, 1002–1006 (2000).

[9]H. Härtel, "A qualitative approach to electricity," Institute for Research on Learning, Report #87-0001, September 1987.

[10]R. Chabay and B. Sherwood, *Matter & Interactions: Electric & Magnetic Interactions* (Wiley, New York, 2002); B. A. Sherwood, and R. W. Chabay, "A unified treatment of electrostatics and circuits," ⟨http://www4.ncsu.edu/~rwchabay/mi/circuit.pdf⟩.

[11]This gradient results in feedback during the transient process, which ultimately produces the steady state in the circuit. The steady current satisfying Kirchhoff laws (charge conservation at the nodes and energy transformation rate, as determined by resistivity of circuit fragments) requires a special distribution of the surface charge producing a particular pattern of the electric field.

[12]These boundary conditions follow from the straightforward application of Gauss's and Stokes's theorems.

[13]Many educators would prefer to simply state that the battery drives conventional current through the battery from − to +, opposite to the Coulomb electric field between the terminals. This statement, however, appears to be highly confusing to a novice. Instead, we could say that the process within the battery causes the separation of electric charges, which could be represented by an *efficient* current in the direction opposite to the Coulomb force within the battery. The efficient current is an imaginary current, which would close the circuit in accord with charge conservation. Actually, no charge makes a closed loop in the circuit, but the constantly occurring redistribution of the atomic charges in the chemical reactions (decreasing the internal electric energy of the products) causes the gathering of electrons on the terminal of the battery. This process, although essentially quantum (tunneling), deserves a qualitative explanation.

[14]Except that there must be some charge on the bends of the wire to turn the electrons.

[15]We should not forget that we discuss only the classical picture in an introductory course. Quantum theory changes the nature of the statement regarding the "inertial movement" of the conduction electrons. The notion of inertial movement of electrons is, however, consistent with the introductory instruction in mechanics.

[16]An explanation of the Poynting vector appropriate for introductory students is usually provided in the context of energy transport by electromagnetic waves. See, for example, D. Halliday, R. Resnik, and J. Walker, *Fundamentals of Physics* (Wiley, New York, 2001), 6th ed., pp. 809–810 or F. W. Sears, M. W. Zemansky, and H. D. Young, *University Physics* (Addison–Wesley, Reading, MA, 1982), 6th ed., pp. 700–703.

[17]At the college level, the Poynting vector is usually introduced without the vector product. See, for example, F. W. Sears, M. W. Zemansky, and H. D. Young, *College Physics* (Addison–Wesley, Reading, MA, 1977), 4th ed., pp. 571–572.

[18]See for example, R. J. Osborne, "Children's ideas about electric current," New Zealand Sci. Teach. **29**, 12–19 (1981); T. A. Borges and J. K. Gilbert, "Mental models of electricity," Int. J. Sci. Educ. **21**(1), 95–117 (1999).

Brooks Inductometer. As late as the 1950s, students studied the theory and operation of alternating current bridges, thus becoming knowledgeable about the use of complex numbers. The Brooks Inductometer was used to produce a variable self-inductance for a single circuit or a variable mutual inductance between two circuits. Inside were four fixed coils and two coils that could be rotated with respect to them with the knob on the top. This model provided a range of 12 to 100 millihenrys. H.B. Brooks and F.C. Weaver of the Natural Bureau of Standards designed the apparatus and it cost $150 in 1920–21, when it was bought by Westminster College. (Photograph and notes by Thomas B. Greenslade, Jr., Kenyon College)