Changing approach to teaching electromagnetism in a conceptually oriented introductory physics course

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An approach emphasizing the complementarity of electric and magnetic fields within a weak relativistic approximation is suggested as a basis for presenting electromagnetism in an introductory university physics course. Within the electromagnetic unification approach, ''magnetic force'' and ''electromagnetic induction'' are each taught in a manner consistent with mechanics from a qualitative relativistic point of view. The Lorentz force and the magnetic flux rule are treated similarly, linking electrical and magnetic phenomena and improving the integrity and self-consistency of the course. The status of Faraday's integral law is discussed and is shown to be of limited validity in this context.

I. INTRODUCTION

In a variety of curricula on electromagnetism within introductory physics courses (IPC) at the college level, ''magnetic force'' is introduced as follows (using standard symbols):

$$\mathbf{F}_m = q[\mathbf{v}\times\mathbf{B}] \tag{1}$$

or, in its nonvectorial and less informative form, as employed in high school,

$$F_m = qvB\,\sin(v,B). \tag{1a}$$

It is then further extended to the Lorentz force that incorporates both electrostatic and magnetic forces,

$$\mathbf{F} = q\mathbf{E} + q[\mathbf{v}\times\mathbf{B}]. \tag{2}$$

Although this material is always presented once the notion of velocity had been discussed as having *only* relative meaning and the Galilean relativity principle established, no comment is usually made regarding the frame of reference in which the velocity in (1) and (2) has to be measured. Moreover, since the principle of indistinguishability of inertial frames of reference (Galileo's relativity principle) is presented as fundamental, this presents a contradiction which one way or another might disturb a curious student or teacher. Indeed, it would appear that (1) and (2) are not invariant in a simple Galilean transformation of velocities $\mathbf{v} = \mathbf{v}' + \mathbf{v}_0$, which would imply that different forces act in inertial frames moving relatively to each other. This conflict is not acceptable, especially in a conceptually oriented physics course. To ignore this problem would mean reducing physics to a cluster of disconnected units of knowledge, contrary to the aims usually proclaimed by physics educators. It may be just those good students who had internalized the constraint of relativity who are most likely to be confused by this inconsistency. The question may be very practical. Suppose a dynamo in a car is considered. What velocity of electrons should be used in the formula to calculate the Lorentz force? Is it relative to the car, to the ground, or to something else? Is a magnet moving and approaching the circuit physically equivalent to the circuit approaching the magnet? Or, when an electrical charge is moving parallel to a current-carrying wire, what velocity should be put into the magnetic force formula (1): relative to the wire, or relative to the charges moving in the wire, or ''relative to the magnetic field?'' Is it appropriate, having stated the observer invariance of physics laws, to introduce the laws of electromagnetism in uncertain frames of reference?[1] Is the laboratory frame of reference the only one possible? Qualitative questions of this kind are often left unaddressed.[2]

As has been shown, students often construct an understanding that, in many ways, is incompatible with the provided formal instruction.[3] The topic of electromagnetism is not an exception.[4] For example, students often revert to an Aristotelian understanding of the force–motion interrelationship in an electric field environment.[5] They often disqualify ''mechanics'' laws (including the most fundamental) as not being applicable to electromagnetism. For example, students discard Newton's third law in an electromagnetic context.[6,7] Therefore, specific efforts are evidently needed to clarify and emphasize the universal character of physical knowledge. The velocity dependence of the magnetic force, as commonly presented today in physics classes, contributes to the fragmentation of physics knowledge.

II. THE MAGNETIC FORCE

Students are commonly confused when questioned about the velocity dependence of magnetic or Lorentz forces.[8] Formally the correct answer is that this is the velocity of the electric charge in the frame of reference in which the magnetic field **B** is experienced. In other words, the velocity **v** of the electric charge q, and the magnetic field **B** are to be measured by the same observer.

Although of fundamental conceptual importance, this question is not usually raised and explanations do not appear in IPC texts. On being asked, students often consider this question difficult and unnecessarily ''philosophical.'' This difficulty in the interpretation of velocity is ultimately related to the understanding of magnetic field. On second thought, however, the dubious knowledge of students has a very respectable parallel. As is well known, the same uncertainty we see in the novice learner of today was that which scientists faced at the end of the era of classical physics. This very problem stimulated dramatic developments in science 100 years ago. Alternative electromagnetic theories were suggested to understand the velocity dependence in (1) and (2). Hertz in 1890 tried to extend the relativity principle of mechanics to electrodynamics, postulating the entire convection of the ether, and Lorentz in 1895 abandoned this principle in favor of the ether remaining at absolute rest. Even earlier, Fresnel (in 1845) had claimed the partial convection of luminiferous ether, and the results of Fizeau's experiment (1851) were interpreted as its solid confirmation.[9] All these theories were replaced by the revolutionary solution provided by Einstein in his special theory of relativity, which unified electricity with magnetism.

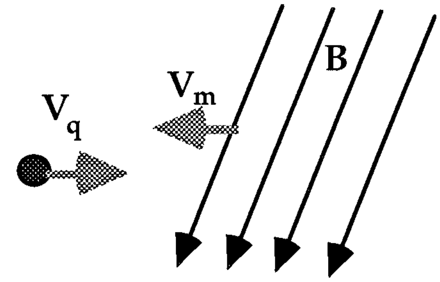

Fig. 1. Novice learners often consider an electric charge moving in a magnetic field as a situation of relative motion (''movement relative to the field,'' ''moving field'' or ''moving with respect to the field''). Textbooks do not always emphasize that this view is conceptually unacceptable.

Since then it has been recognized that electricity and magnetism present different facets of one and the same physical entity. One would expect the relativistic approach to be unavoidable in instruction aimed at a conceptual understanding of electromagnetism, simply because there is no competitive theory. In spite of this, IPC textbooks commonly continue to present electromagnetism in a ''traditional,'' pre-relativistic way, sustaining crucial conceptual uncertainties. Not specifying the frame of reference in (1) and (2) (where the velocity, not to mention the magnetic field, is supposed to be measured), is an example of such instruction.[10] There are exceptional authors[11] who comment (often in optional material) on the observer invariance of a force exerted on an electric charge in the presence of an electric current, or on the observer dependence of magnetic interaction.[12] The text by Purcell,[13] which cannot be identified as introductory, does not include such an uncertainty.

Some authors,[14] when accounting for the magnetic force, do treat the velocity as relative,

$$\mathbf{F}_m = q[(\mathbf{v}_q - \mathbf{v}_m) \times \mathbf{B}_m],$$

where $\mathbf{v}_q$ is the velocity of the charge, $\mathbf{v}_m$ the velocity of the magnet, and $\mathbf{B}_m$ is the magnetic field. This means the velocity **v** in the Lorentz force is reduced to the one relative to *the magnet* or, to the current-carrying *wire* creating the magnetic field. Even in highly respected texts one can find claims of the charge's relative motion ''with respect to the magnetic field.''[15] Such expressions as ''The students must recognize that as the particle moves it carries its electric and magnetic fields with it''[16] can be easily misunderstood, contrary to the author's intention to convey very different ideas. Practicing teachers often observe such conceptually unacceptable ideas being easily adopted by students. Physicists would be amazed at the theories of magnetic field ''moving with the magnet'' or ''remaining stationary'' which were considered legitimate in electrical engineering, probably, for very pragmatic goals.[17] This misconception was addressed by Feynman in his lectures.[18] Teachers' metaphors (or literal interpretations) of ''movement relative to the field,'' or ''moving field'' (Fig. 1) often resonate with students general tendency to reify, ‘‘materialize,’’ abstract physical concepts.[19]

Though for low velocities one may not be making a significant numerical mistake by using this metaphor,[20] the adoption of this use of ‘‘relative velocity’’ is educationally undesirable. In the formally correct approach, changing the observer cannot be reduced to the application of the relative velocity (by velocity transformation) alone but requires field transformation which (and this is a fundamental claim of the special theory of relativity) incorporates both electric and magnetic fields. Finally, the two-part Lorentz force (2) should be applied, and not only its magnetic part (1).

A pedagogically important point lies in the inherent unity between electric and magnetic fields as facets of the same physical entity. This fact comprises a conceptual framework at *all* velocities. Some students might think that in electromagnetism, as in mechanics, when velocities are much lower than that of light, the case belongs to Galileo–Newtonian physics in which one can consciously neglect relativistic effects. However, this view is essentially incorrect; no matter how high or low the charge velocity is, the magnetic force *completely* disappears for an observer moving with the charge. Thus the separation, common in mechanics, between ‘‘classical’’ and ‘‘relativistic’’ domains is not valid in electromagnetism and a conceptually correct and consistent understanding of electromagnetic phenomena can be reached only within a relativistic framework.

The question arises whether a consistent knowledge of electromagnetism lies in the realm of advanced courses and, as such, should be ignored in an IPC. If not, we face a challenge: to find an appropriate form of this knowledge which would be both correct and also simple enough to be incorporated into an IPC. It is our view that a construction of relevant conceptually correct knowledge can be reached by a majority of college students, even by those who may never go on to learn relativistic physics.

Let us consider the Lorentz force in two inertial frames: S, where the magnet is stationary and S', where the charge is stationary. To prepare a possible simplification, we start within a fully relativistic description. Suppose that an observer, S, is subjected to a homogeneous magnetic-field $\mathbf{B}$ along the z axis [Fig. 2(a)]. That is, the electromagnetic field is

$$\{\mathbf{E}(0,0,0);\mathbf{B}(0,0,B)\}. \tag{3}$$

The Lorentz force that acts on the charge moving with a velocity $\mathbf{v}$ along the x axis is

$$\mathbf{F}(0,-qvB,0). \tag{4}$$

An observer, S', [Fig. 2(b)], moving with the charge, could experience fields as follows:

$$\begin{cases} E'_x=E_x, \quad E'_y=\gamma(E_y-vB_z), \quad E'_z=\gamma(E_z+vB_y) \\ B'_x=B_x, \quad B'_y=\gamma\left(B_y+\dfrac{v}{c^2}E_z\right), \quad B'_z=\gamma\left(B_z-\dfrac{v}{c^2}E_y\right), \end{cases} \tag{5}$$

where

$$\gamma=(1-v^2/c^2)^{-0.5}. \tag{5a}$$

The fields in S' are

$$\mathbf{E}'=(0,-\gamma vB,0), \quad \mathbf{B}'=(0,0,\gamma B). \tag{6}$$

Therefore, the force acting on the charge, as measured by S', is

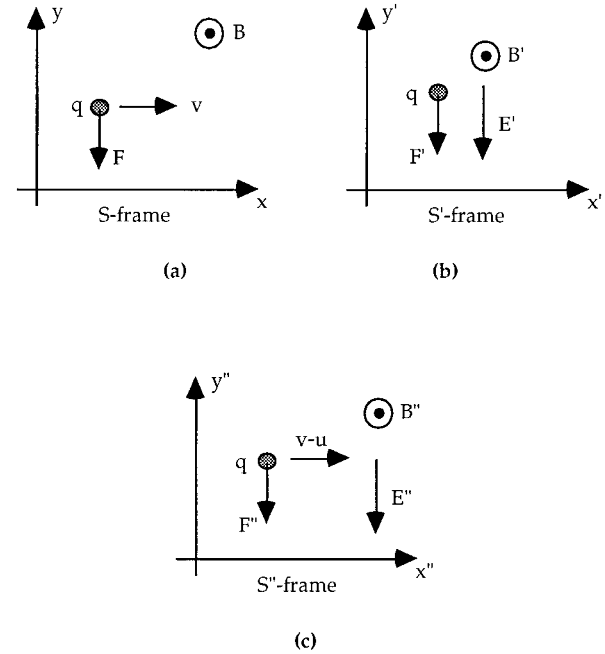

Fig. 2. (a) An electric charge q moving with velocity v in a magnetic field B as measured in frame S; (b) the same charge q when considered in the frame S' moving with velocity v to the right, experiences the electric field $\mathbf{E}'$ and magnetic field $\mathbf{B}'$; (c) the same charge q when considered in the frame S'' moving at arbitrary velocity u to the right, relatively to frame S, experiences electric field $\mathbf{E}''$ and magnetic field $\mathbf{B}''$.

$$\mathbf{F}'=(0,-\gamma vqB,0)=(0,qE',0), \tag{7}$$

which is in accord with the relativistic dynamics[21] ($F_y=F'_y/\gamma$). This result is quite remarkable. The force which is registered in the S frame as purely magnetic is purely electric in the S' frame with nothing physically changed, besides the observer. This point could be used to elucidate the idea that electric and magnetic fields are facets of one physical entity, like a single object being viewed from different angles. One can also see here that the issue of the force’s velocity dependence cannot and should not be reduced to the *relative* velocity of the charge and the magnet with respect to each other (which is not merely the difference between their velocities). Thus, only with the help of relativity, the paradox of disappearing force is resolved.

Most (yet not all[22]) physics educators see a complete relativistic treatment as inappropriate within an introductory course. The textbook by Purcell,[23] which manipulates the full set of Lorentz transformations (5), is more commonly used in advanced physics courses. However, the underlying unity of electric and magnetic fields (forces) and their conversion of one into the other for different observers, should be the focus of conceptually oriented instruction *even at the introductory level*. The Lorentz transformation in its low velocity limit (weak relativistic approximation—WRA) might provide a pedagogically appropriate basis for this goal. WRA includes only a linear velocity dependence and reduces the full set (5) into the form,

$$\begin{cases} \mathbf{E}'=\vec{E}+[\mathbf{v}\times\mathbf{B}] & \text{(8a)} \\ \mathbf{B}'=\vec{B}-\dfrac{1}{c^2}[\mathbf{v}\times\mathbf{E}]. & \text{(8b)} \end{cases}$$

Although expressions (8) are of no greater mathematical complexity than (1) and (2), they already manifest the hybrid nature of the two fields.[24] Here one can draw another analogy, with a transformation of vector coordinates in a transition between different frames of reference. Furthermore, an important advantage of the WRA is the invariance of force (2) in changing the inertial frame. This fact, which holds only within WRA (and not at high velocities), is in keeping with students' intuition. The observer invariance of force seems plausible to students and, as such, might be pedagogically favorable. Both fields appear as complementary in their contributions to the experienced force, the ratio of the contributions varies from one observer to another but the resultant effect remains, presenting a kind of conservation. A tendency to conserve physical entities is strongly entrenched in children's operational knowledge constructed by individuals spontaneously, in an early stage of their cognitive progression.[25]

To introduce transformation (8) within an IPC, one may start with the same setting of the charge moving with respect to a stationary magnet [Fig. 2(a)]. The fields in frame S are given by (3) and the Lorentz force on the charge by (4). If one follows a historical sequence, the magnetic force $F_L = qvB$ on a charge carrier q can be derived from the Ampere force $F_A = \mathrm{IBL}$ (common notations) exerted on a current-carrying wire. In the frame S', however, the charge is stationary, and hence, does not experience any magnetic force [Fig. 2(b)]. Having established that force is invariant and that the charge experiences no magnetic force, only an electric field can account for the force on the charge in frame S'. Evidently, this field has to be $\mathbf{E}'(0,-vB,0)$ in order to cause the same force. The resultant force invariance is then interpreted as demonstrating the complementary and interchangeable natures of electric and magnetic fields. The magnitude and direction of the electric field in S' is exactly such that the change in electric force compensates for the disappearance of the magnetic force. Thus an explicit expression for the electric field $\mathbf{E}' = [\mathbf{v} \times \mathbf{B}]$ is obtained and can be extended to (8a). The students may be further encouraged to consider an arbitrary frame S'' moving with an arbitrary velocity, $\mathbf{u}$, in order to check the generality of the inference [Fig. 2(c)]. The suggested invariance of force implies that the fields $\mathbf{E}''(0, -uB,0)$ and $\mathbf{B}''(0,0,B)$ should be registered in the frame S''. In S'' the charge and magnet move with a relative velocity of $\mathbf{v}-\mathbf{u}$ and the Lorentz force is given by

$$\mathbf{F}'' = q\mathbf{E}'' + q[(\mathbf{v}-\mathbf{u}) \times \mathbf{B}''], \tag{9}$$

which yields

$$F'' = -quB + q(v-u)(-B) = -qvB, \tag{10}$$

confirming that the force invariance

$$\mathbf{F}'' = \mathbf{F}' = \mathbf{F} \tag{11}$$

is in accordance with the transformation (8a). This line of thought implies the important idea that a magnet creates more than just a magnetic field. It creates something that appears as a magnetic field to the observer who perceives the magnet as stationary, but, to an observer who sees it moving, the magnet creates an electric field as well. Together, both fields complement each other, giving rise to the same dynamic reality defined as the Lorentz force. To the student, electric and magnetic forces now beg to be synthesized into a new complex expression: Although each of them varies separately from observer to observer, their sum (2) is invariant and hence represents one physical concept. This treatment is in keeping with the Galilean relativity principle and extends it beyond mechanics. It also resolves an apparent discrepancy between ''mechanics relativity'' (coordinates, velocity, Newton's laws) and electromagnetism (the velocity dependence of the magnetic force); thus the mechanics and electromagnetism units of the curriculum become consistent. We have managed all this without a complete relativistic treatment and without even mentioning the invariance of the speed of light or discussing subtle effects of relativistic kinematics.

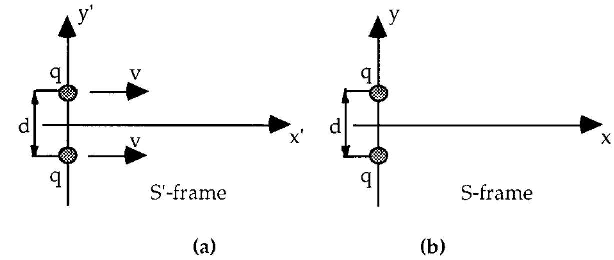

Fig. 3. (a) Two electric charges q moving with velocity v, as viewed in frame S'; (b) the same charges when considered in frame S moving with the velocity v to the right.

The discussion of a single-charge-in-a-field can be complemented with the example of a pair of identical electric charges moving with velocity $\mathbf{v}$ [Fig. 3(a)]. Here, we cannot expect a first-order correction to the electrostatic-force: the magnetic field created by each of the charges is proportional to velocity, and the correction in (8a) provides additional v proportionality. Altogether, this produces the familiar result of the magnetic correction being proportional to v^2/c^2; which goes beyond the approximation (WRA) that we have adopted.[26] (This result immediately allows us to estimate the ratio of electric and magnetic forces between the same charges.) However, to be consistent, one can apply the field transformations to expose the limits of the WRA.

Consider two charges separated by a distance d and moving at identical velocities v, relative to observer S'. S' can account for the electric and magnetic fields, created by the first charge at the location of the other, using their common classical forms (which hold within the WRA[27]),

$$\mathbf{E}' = \left(0,-\frac{1}{4\pi\epsilon_0}\frac{q}{d^2},0\right) \quad \text{(Coulomb law)}, \tag{12}$$

$$\mathbf{B}' = \left(0,0,-\frac{\mu_0}{4\pi}\frac{qv}{d^2}\right) \quad \text{(Biot–Savart law)}. \tag{13}$$

The force between the charges is given by summing the magnetic and electric forces,

$$\begin{aligned}\mathbf{F}' &= \left(0,-\frac{1}{4\pi\epsilon_0}\frac{q^2}{d^2}+\frac{\mu_0}{4\pi}\frac{q^2v^2}{d^2},0\right)\\ &= \left(0,-\frac{1}{4\pi\epsilon_0}\frac{q^2}{d^2}\left(1-\frac{v^2}{c^2}\right),0\right)\\ &\approx \left(0,-\frac{1}{4\pi\epsilon_0}\frac{q^2}{d^2},0\right).\end{aligned} \tag{14}$$

To be consistent with the WRA and to keep the electromagnetic force observer-invariant, the v^2/c^2 correction of magnetic origin[28] should be neglected. On the other hand, applying the transformation (8a) to (12) and (13), one accounts for the fields in the frame of reference S [Fig. 3(b)], where the charges are stationary,

$$\mathbf{E}=\left(0,-\frac{1}{4\pi\epsilon_0}\frac{q}{d^2},0\right), \tag{15}$$

which implies a pure electrostatic interaction,

$$\mathbf{F}=\left(0,-\frac{1}{4\pi\epsilon_0}\frac{q^2}{d^2},0\right), \tag{16}$$

in accord with (14) and the idea that the interaction which is pure electrostatic in one view is complex electromagnetic, in the other.

In fact, treatment of the interaction between two moving charges poses a dilemma for the educator. If one keeps the v^2/c^2 term in (14) and claims that the EM force is observer-dependent, the whole edifice of classical physics crumbles. One can proceed by showing the failure of Newton's third law[29] or conservation of momentum, for a general configuration of two moving charges.[30] Students are then referred to the course on relativity to settle these problems. Chabay and Sherwood[31] used the demonstrated observer-dependence of the force to introduce the relative nature of time.[32] In fact, this explanation could spark a full relativistic discussion. One should refrain from doing so, however, if one has decided to remain within the force invariance of the WRA. Being supported by intuition, this framework can present the unification of electric and magnetic fields in simpler terms without touching on the space-time relativistic phenomena. Such an approach could be useful for teachers who prefer to focus only on electromagnetic aspects in their course and to avoid revising mechanics.

After (8a) has been established, one might complete the revision of the relationship between the fields with the question of what happens to the magnetic field (8b). Here, another situation can be considered. Suppose a point charge q is stationary in S. In another frame S', moving with velocity $\mathbf{u}$ relative to S, the charge q moves with the velocity $-\mathbf{u}$, creating a current and hence a magnetic field. This is commonly described by the Biot–Savart law, which was obtained historically in its integral form, for a long straight wire, and then reduced by Laplace to an elementary current Idl.[33] Reducing it even further to a single moving charge,[34] this law implies a magnetic field $\mathbf{B}'$ in the S' frame,

$$\mathbf{B}'=-\frac{\mu_0}{4\pi r'^3}\,q[\mathbf{u}\times\mathbf{r}']. \tag{17}$$

Here $\mathbf{r}'$ stands for the radius vector from the location of q to the point where the magnetic field is measured.

Meanwhile in frame S (stationary charge) only an electric field is perceived,

$$\mathbf{E}=\frac{q}{4\pi\epsilon_0 r^3}\,\mathbf{r}. \tag{18}$$

A closer look at (17) reveals a way to re-express the magnetic field in S' in terms of the electric field in S, (18). This is useful when introducing the relationship between the two fields (8b). Taking advantage of the fact that the WRA keeps spatial variables observer invariant, one obtains

$$\mathbf{B}'=-\epsilon_0\mu_0[\mathbf{u}\times\mathbf{E}]=-\frac{1}{c^2}[\mathbf{u}\times\mathbf{E}]. \tag{19}$$

This, result reveals another facet of the intimate relationship between electrostatics and electrodynamics. Equation (19) complements the back-and-forth relationship between electric and magnetic fields. Although the correction obtained for the magnetic field can be neglected in the particular approximation[35] that we are using its significance is conceptual; the smallness of the correction justifies the assumption that the magnetic field does not change numerically between frames.

The idea that fields are observer dependent is no more obvious than the relativity of time or length. Prior to instruction, physics freshmen assume the observer-independence of fields without a second thought (which means, in our notations),

$$\begin{cases}\mathbf{E}'=\mathbf{E}\\ \mathbf{B}'=\mathbf{B}.\end{cases} \tag{20}$$

The new approach may start from this formal framework of students' naive ideas about field conservation and proceed through its modification into a more appropriate form. Using a constructivist-type class discussion (or a Socratic teacher–student dialogue[36]), physics teachers may provoke students dissatisfaction with their naive idea of ''field invariance'' by discussing simple cases rather than a full relativistic theory. Thus the modification of (20) into (8) is technically feasible and more suitable within an IPC. The conservation of force (to obtain the **E** transformation), and the link between the Coulomb and Biot–Savart laws (to address the **B** transformation), seem to meet the criteria for meaningful learning, they are intelligible, plausible and fruitful.[37] The suggested approach is oriented on the construction of new knowledge, instead of its adoption. It emphasizes the velocity dependence of the magnetic force (1) instead of ignoring it. The field transformation (8), though only an approximation of the complete Lorentz transformation (5), has other advantages besides its compact form. Ultimately, they are those of the theory of relativity over pre-relativistic physics: symmetry, integrity, consistency, completeness and invariance. All these are certainly of educational value and as such should be exposed to students already in the introductory course. In contrast to the IPC, advanced courses in electromagnetism are usually deductive.[38] The WRA could be derived there, if at all, from the Lorentz transformation but it does not play a significant conceptual role at that level of instruction.

Finally, it is useful to differentiate between the suggested unification of electric with magnetic fields and other types of relationship between physical entities. Physics regularly links and causally connects physical entities. For instance, Newton's second law relates the resultant force on a body to its acceleration. Electric and magnetic phenomena were related by Maxwell's set of equations. Einstein's linking of electric and magnetic fields is of a different nature. The field transformations show us that electric and magnetic fields present the exact same entity, which can *simultaneously* appear to coexist in different proportions to each other depending on one's frame of reference. It is as if facets of the same solid are observed in different perspectives (Fig. 4). Of course, this analogy between literal observation and the ''observation'' of the electromagnetic field (force) is limited and

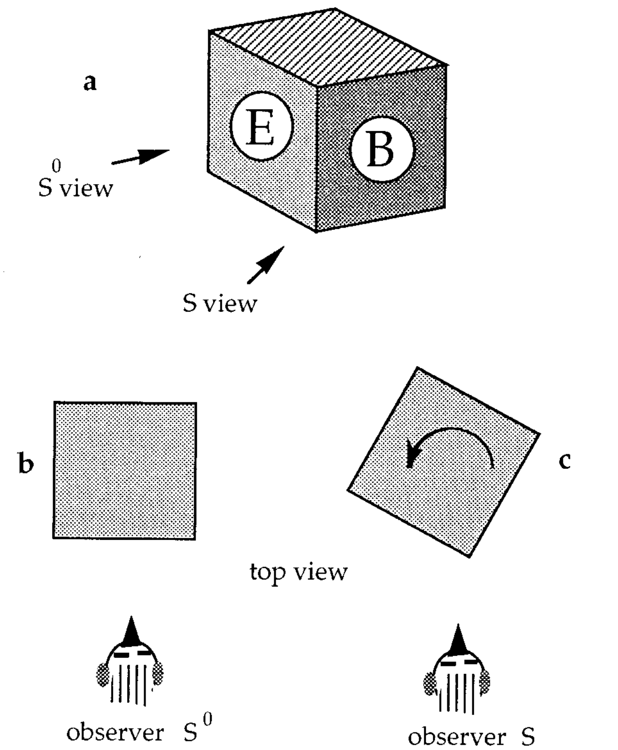

Fig. 4. (a) A way to display the unity of the electric and magnetic fields. Two facets of a cube represent the fields. The observed combination of fields depends on the ''point of view'' (state of motion). (b) S^0 (a stationary electric charge), facing a front of the cube, observes only an electric field. (c) S (a moving charge), viewing slightly from a side, observes a combination of electric and magnetic fields. The change of view corresponds to an effective turn of the cube.

not quantitative. Nevertheless, it can be valid for instruction as conveying one of the major ideas in relativistic electromagnetism.

III. ELECTROMAGNETIC INDUCTION

The unification of electric and magnetic fields, when established in the unit on magnetic force, gains even more importance if it is extended to electromagnetic induction. Indeed, just like the velocity dependence of the magnetic force, the *relative* motion of a conducting rod (or circuit) and a magnet may cause confusion in understanding electromagnetic induction. Despite the obvious symmetry of the situations, we account for them by induced electromotive forces of different kinds. Does the phenomenon itself depend on the frame of reference of the observer? Formal physics would answer that it does not (at nonrelativistic velocities[39]). This claim is in keeping with intuition of ''force invariance'' within the WRA. In fact, the above-mentioned asymmetry served as a starting point for Einstein's discovery of the special theory of relativity.[40]

Historically, Faraday in 1831 understood electromagnetic induction in the same way as many of our students do nowadays: He believed that it was caused by the cutting of ''magnetic lines of force'' by a conductor moving *relative* to a magnet (Fig. 5).[41] A more complicated picture was suggested by Faraday to understand the general case. He distinguished between magnet translation, where the lines of force travel with the magnet, and magnet rotation where lines of force

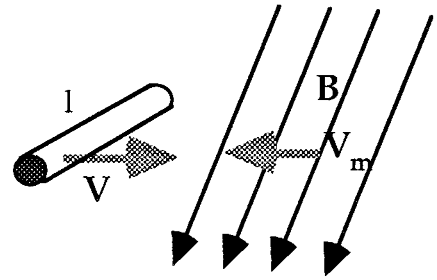

Fig. 5. Faraday considered the magnetic field (''lines of magnetic force'') as a material reality and so considered the conductor moving in a magnetic field as a case of relative movement where the conductor cuts the lines. Novice learners will often employ this same view.

remain stationary in space.[42] This understanding of the magnetic field as a material entity, which can move or remain stationary independent of the magnet itself, is incompatible with the currently adopted paradigm of electromagnetism. Nevertheless this view remained in use many years after it was theoretically discarded.[43] The old ''relativistic'' paradigm, which considers only the relative motion of the magnet and conductor but not their relation to the observer, is still considered appropriate to represent electromagnetic induction so long as no formalism is applied.[44]

In an IPC, as within Maxwell's theory of electromagnetism, it is normal to differentiate between two cases in which we observe induced electromotive force (emf). At first, physicists were struck by a puzzling asymmetry between a moving magnet and a moving charge (or conductor) that exists within Maxwell's formalization.[45] Indeed, the induced emfs are explained and identified differently whether the object at rest is the magnet, ''motional emf'' or the conductor, ''the emf due to the changes in magnetic field.''

Observer S [Fig. 6(a)] detects an induced emf and explains it as the result of a magnetic force on moving charges. Observer S' [Fig. 6(b)] detects the *same* force but explains it as a result of an electric field in keeping with Maxwell's equation,

$$\nabla\times\mathbf{E}=-\frac{\partial\mathbf{B}}{\partial t}, \tag{21}$$

which implies a curly, non-Coulombic E field due to temporal changes in the magnetic field. Both the laws for magnetic force (1) and for electric field (21) are differential (they include only the local values of fields, or their derivatives) and so provide different mechanisms for the induced emf in each of the two cases. The curly, non-Coulombic electric field exists whether or not a conductor is present. A conductor merely provides an opportunity for the curly field to manifest

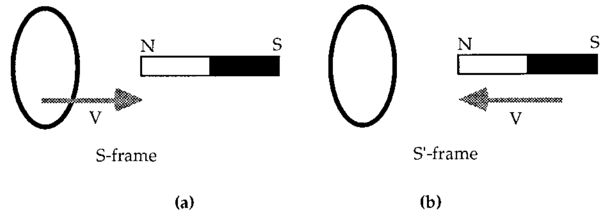

Fig. 6. (a) A conducting loop moving with velocity v to the right, towards a stationary magnet, as viewed in frame S; (b) the same situation when considered in frame S' moving with velocity v with the loop.

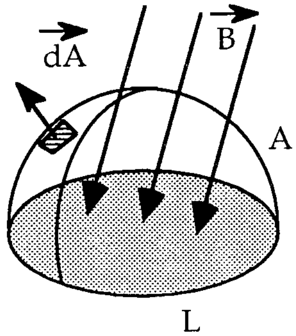

Fig. 7. A spatial loop L is covered by surface A, over which the integral of magnetic field B is calculated, yielding the flux Φ, relevant for Faraday's law.

itself in an electric current (or voltage). Equally, it is meaningless to talk about motional emf in absence of an electric charge. These emfs seem to be different physical phenomena. Yet Faraday saw both cases of emf induction as identical phenomena and subject to the same law, since for him the electromagnetic induction was a matter of ''intersecting of magnetic curves'' which happens during the *relative* motion of the magnet (together with its ''magnetic curves'') and the conductor.[46] The distinction between the two processes came only with Maxwell's theory.

Purcell, in his advanced course, generalizes the cases of stationary and moving magnets further by showing that they can be reduced to one law.[47] Within it, an induced emf is related to the rate of change of magnetic flux through an area A which caps the considered circuit path L (Fig. 7). Observers in relative motion would write it as follows:

$$\mathscr{E}_{\text{ind}}=\frac{d\Phi_B}{dt},\qquad \Phi_B=\int\int_A \mathbf{B}\cdot\mathbf{dA}\quad (S \text{ frame, moving conducting loop}) \tag{22a}$$

or

$$\mathscr{E}'_{\text{ind}}=\frac{d\Phi'_{B'}}{dt'},\qquad \Phi_{B'}=\int\int_{A'} \mathbf{B}'\cdot\mathbf{dA}'\quad (S' \text{ frame, stationary conducting loop}), \tag{22b}$$

which coincide, for low velocities, because all spatial–temporal relativistic corrections are too small and the magnetic field is practically frame invariant. This integral statement is known as Faraday's law of induction, though Faraday himself neither wrote nor defined it this way.[48] How to understand electromagnetic induction was much debated until the interconvertible nature of electric and magnetic fields was introduced by Einstein's Special Theory of relativity.[49] From its perspective, identification of an induced emf is not absolute but depends upon the observer who experiences a particular configuration of electric and magnetic fields in his frame of reference.

Although the coincidence of (22a) and (22b) provides a quantitative (if not qualitative) justification for employing magnetic-circuit *relative* motion symmetry, it does not stretch beyond this. The use of the metaphor ''moving field'' has been fundamentally reconsidered since 1905.[50] In spite of this, IPC textbooks often employ the metaphor of ''cutting field lines'' in the form which does not prevent its obsolete interpretation.[51] Rarely is the conceptual bridge between the two kinds of induced emf explained.[52]

Moreover, there is a difference between the status of the differential law and that of the integral one. Although Maxwell's equation (21) is sometimes called ''the Faraday law in its differential form'' and equated with it, these two are not equivalent. By virtue of Stoke's theorem, the integral law (22) can be reduced to the differential statement (21) but only in the case of a stationary circuit.[53] Therefore, Purcell's statement that the laws are *entirely* equivalent is imprecise,[54] as the motional emf is not included in (21) but may be in (22). Some of those who tried to obtain a truly equivalent differential form for the Faraday law obtained[55]

$$\nabla\times\mathbf{E}=-\frac{\partial\mathbf{B}}{\partial t}+\nabla\times(\mathbf{v}\times\mathbf{B}), \tag{21a}$$

which could confuse even more as if suggesting a modification of the Maxwell equation (which it is not, as the **E** and **B** in (21a) are to be measured in different frames of reference). Probably similar intentions brought Cohn[56] to state two integral laws of induction, the flux rule (22) and, independently, the rule for motional emf,

$$\mathscr{E}_{\text{ind}}=Blv\quad (\text{with usual notations}). \tag{23}$$

The path of integration in the integral law (22) might present another point of misinterpretation. It is natural to identify this path with a material circuit to obtain the induced emf in it. However, subtle effects of integration, which do not reflect any new physics, may cause confusing results when applying the integral law. We will illustrate this below.

In presenting electromagnetic induction within an IPC, some authors[57] begin with the magnetic force on a moving charge in a magnetic field. They interpret this force as a manifestation of the motional emf in a rod moving through a magnetic field, (23). Thus Faraday's law (22) appears to be a generalization of a microscopic phenomenon of a magnetic force acting on a charge. Then, the effect of the induced electric fields is taken to be explained based on the integral law. It was an innovation to start from the induced, non-Coulombic electrical field and to generalize to Faraday's integral law.[58]

Most authors take a different approach. They first present the integral law (22) as an empirical discovery by Faraday.[59] Once this law is established, the emf for a conducting rod moving in magnetic field (23) can be deduced. The induced emf by a magnet approaching a stationary circuit is also explained as a result of temporal changes in the magnetic field and hence in the magnetic flux. Such an approach is closer to the historical sequence of events, but is it conceptually satisfactory? The false impression might emerge that the two phenomena of induced emfs are derivatives of Faraday's law (22), which appears fundamental and all-encompassing. In fact, as far as one considers the theory of electromagnetism, the situation is different. From the integral law (also known as the ''flux rule''), one can obtain striking ''paradoxes,'' which require an extra effort of interpretation based on differential laws in order to be resolved.

A clear example of such a case was provided even by Faraday himself ''Faraday's disc'' does not need any change in magnetic flux to produce an induced emf [Fig. 8(a)]. In

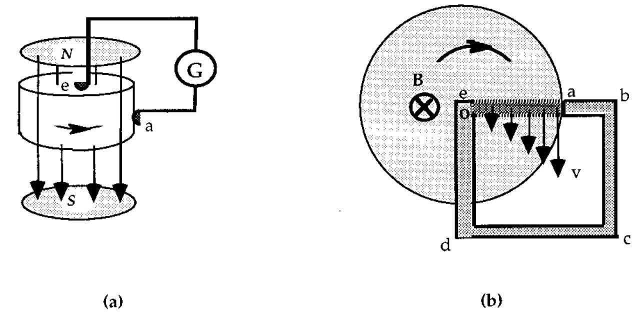

Fig. 8. (a) A schematic view of Faraday's disc generator; (b) an electrical circuit (**oeabcd**) in the disc generator.

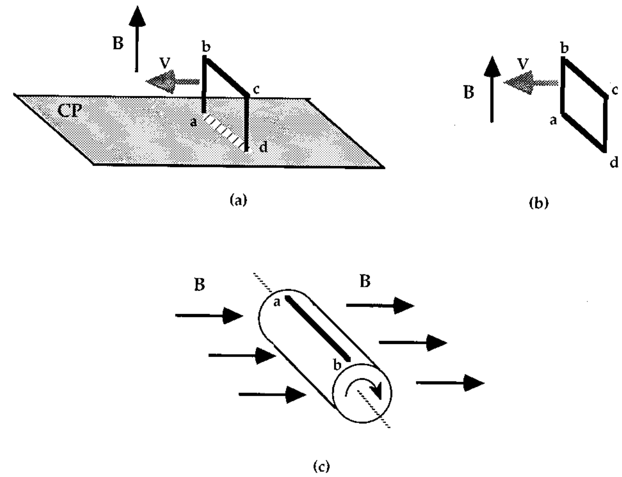

Fig. 9. (a) Rectilinear geometry equivalent to the case of a disc generator; (b) the case of zero flux through the loop and a non-zero voltage in it; (c) the conducting rod ab has been mounted on a wooden cylinder rotating in a homogeneous magnetic field. Despite the absence of any change in magnetic flux (the loop is undefined), an electrical current flows in the rod.

this device a solid conducting disc rotates in a constant uniform magnetic field parallel to the disc axis. Though the relative motion of the disc and the magnet is present, the magnetic flux through the disc does not change, $d\Phi_B/dt = 0$. However, Faraday demonstrated that a potential difference is developed between the center and the periphery of the disc: a curious situation where Faraday's law does not account for Faraday's generator.[60]

In contrast, a microscopic (differential) approach which specifies the magnetic force $q[\mathbf{v}\times\mathbf{B}]$ applied on charges in the moving disc, easily provides a complete account for the created emf. Figure 8(b), schematically represents the electrical circuit. From the whole circuit (**abcde**), we need only consider a radial ''rod'' in the rotating disc which instantaneously connects the two brushes, brush **e** in the center, with brush **a** at the periphery. As the disc rotates, all its charges possess different tangential velocities perpendicular to the radius **ea**. The other velocities of electrons in the **abcde** circuit (drift and thermal) are not relevant. The tangential velocities of electrons are subject to the magnetic force pushing them radially outward. This push is interpreted as an induced emf, spread across the disc and causing a radial potential difference. While a straightforward use of magnetic force easily accounts for this effect,[61] too strict a use of the integral law (22) fails because there is no change in the magnetic flux linked to the circuit. The advantage of the microscopic law, in this case, is evident.

Although an appropriate conceptual discussion of ''Faraday's disc'' might be very instructive, it rarely appears in IPC texts. Commonly, it appears in the end-of-chapter exercises,[62] which means that its high pedagogical potential is realized only occasionally. Comments advising careful choice of the path of integration which ''should incorporate the motion of the disc,''[63] are obscure and of little help to the learner. The recommended method of following the change in an arbitrary chosen sector area, though it leads to the correct answer, seems to be a trick and does not explain much. Indeed, there would be no induced current in a closed conducting loop of any shape moving *as a whole* through a homogeneous magnetic field. The failure of the integral law (22) in case of ''Faraday's disc'' is not related to circular motion. It lies in the fact that of the two fragments **ea** and **dc** [Fig. 8(b)] of the conducting loop, only one is in motion and the other is stationary. One can equally observe the same phenomenon in its rectilinear geometrical analog. Consider a U-shaped conducting rod moving in a homogeneous magnetic field [the U plane is parallel to the field, Fig. 9(a)]. The emf induced in the upper side **bc** causes an electric current in the loop **abcd**. The circuit is closed by the **ad** path in the conducting plate **CP** which remains stationary. The flux through the loop remains zero and the flux rule fails to account for the induced current.

In the setting of Fig. 9(b) (a conducting loop **abcd** moving through a homogeneous magnetic field), the flux rule cannot account for a potential difference across the sides **bc** and **ad**—the loop is polarized, and the magnetic flux through the loop remains zero. In contrast to Fig. 9(a), there is no current in this case because the potential difference established across **bc** is the same as that across **ad**. In Fig. 9(a) the charges in the conducting path **ad** do not move with the U-shaped rod so the potential difference across **bc** remains unbalanced.

To emphasize the point, another device can be considered in which an induced emf and an induced electrical current (both oscillating) are present in the absence of a conducting loop or any magnetic flux linked to it. A conducting rod **ab** is mounted on a wooden (insulator) cylinder [Fig. 9(c)]. When the cylinder rotates in a homogeneous magnetic field, an induced current circulates within the rod. The magnetic flux rule is not applicable. Similarly, there is no way to apply the flux rule to account for the induced emf and current in any rod antenna as the loop is undefined. The microscopic law (21) must be applied. We have demonstrated that a change in magnetic flux is not a necessary condition for electromagnetic induction. But is it sufficient?

The question of sufficiency was addressed by Feynman. His example complements the discussion of the validity of the relationship of the induced emf to the rate of magnetic flux change. He described a contrivance in which a significant change in the magnetic flux linked to a circuit does not cause a corresponding induced emf.[64] The geometry of the setting is not simple. Two metal plates **m** and **n** can rotate about hinges O_1 and O_2 [Fig. 10(a)]. The whole construction is flexible and the hinges are not fixed in space. The plates touch each other at point **c**, establishing a closed circuit. The circuit is flat, and the magnetic field is at right angles to the plates. When the plates rock back and forth one over the other, the contact point **c** changes its location, and the area

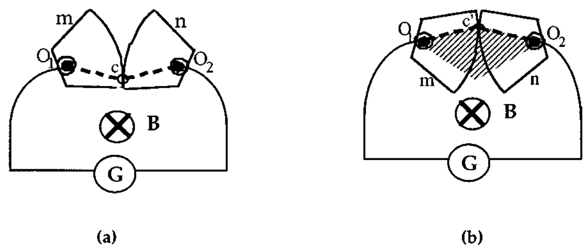

Fig. 10. Feynman's example in which there is a discrepancy between the rate of change in magnetic flux through a circuit loop and the induced emf created in it. The shaded region in (b) represents the relatively large change in the area containing magnetic flux compared with (a).

contained by the circuit varies. Figures 10(a) and 10(b) show two point **c** locations. The shaded region indicates the change in the area between the two positions. Feynman points out that though the plates may move very slowly (small magnetic forces on electrons and, hence, a small motional emf), a substantial change in the swept area may accompany the movement of the contact point. This is because the circuit is closed each time through a new touching point, which can move with any speed! Thus, the change in the loop area containing the magnetic field may cause a large rate of flux change $\Delta\Phi_B/\Delta t$. The discrepancy between a small induced emf, which is deducible from the small magnetic force on the electrons in the plates, and a large change in magnetic flux $\Delta\Phi_B/\Delta t$ due to the quick change in the swept area, determined by the movement of the touching point between the plates, demonstrates the failure of statement (22). Lenz's law which describes the direction of the induced current, however, remains valid because, despite the discrepancy between the velocity of the touching point and the velocities of the electrons, both velocities are in the same direction (Fig. 11).

These examples illustrate the limited validity of the integral law (22). It may be helpful in certain situations, but it is neither of explanatory nor of general power. The microscopic treatment based on forces (fields), on the other hand, is both fundamental and explanatory and, as such, is preferable for educational purposes. Thus the appropriate implementation would be to switch the explanations as we provide. For example, almost all authors present a metal rod sliding along conducting rails in a magnetic field and calculate the emf by applying the flux rule. Instead, although the answer remains the same ($\mathrm{emf}_{\mathrm{ind}}=Blv$), a more instructive treatment would draw on the magnetic force in the moving rod. There is much truth in Feynman's claim that in case of any confusion one should return to differential laws, (2) and (21), which are fundamental. The field (force) approach uncovers the physics involved, whereas the integral law (22) is a rule which can fail.

In spite of the limitations on its generality, the integral magnetic flux law does remain important conceptually. We see its pedagogical role to be similar to that of the Lorentz force. It conceptually unifies two different kinds of electromagnetic induction, demonstrating that the difference between them is not absolute, but that they are complementary in a relativistic sense, much like the identification of force as magnetic or electric. It is up to the physics educator to ensure that by unifying these two complementary phenomena, the flux rule does not take their places, or that of their corresponding basic laws, in the hierarchy of physical knowledge established within the IPC.

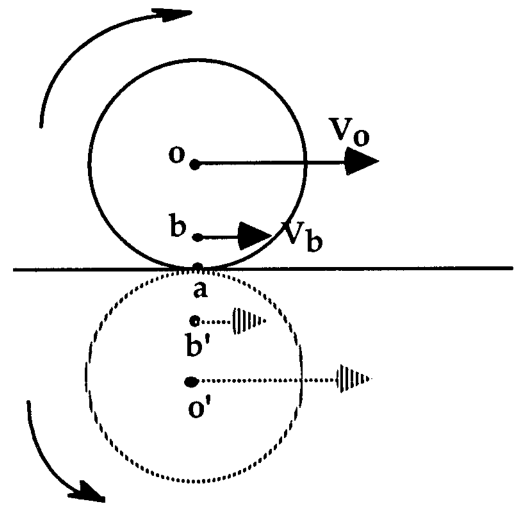

Fig. 11. Two wheels rolling on top of each other, demonstrating the discrepancy between the velocities of physical points in the wheels and their joint point of contact. In Feynman's setting, velocity $\mathbf{V}_0$ of the touching point **a** (not to be confused with physical points at contact which are instantaneously stationary) determines the area swept out by the current path. Physical points **b**, **b**′ move slower. Their velocities, $\mathbf{V}_b$ determine the Lorentz force on the electrons and hence, the induced emf. Lenz's law is preserved because velocities $\mathbf{V}_0$ and $\mathbf{V}_b$ are in the same direction.

IV. FINAL COMMENTS AND CONCLUSION

We advocate inclusion of a simplified relativistic approach to the unit of electromagnetism within an IPC, because it would establish consistency with mechanics and give integrity to physics instruction. This effort would be ideologically similar to that of conceptual bridging between electrostatics and electrical circuits.[65] The suggested approach would encourage the construction of a synthesized view that electric and magnetic fields are two complementary facets of one entity. A WRA could provide the appropriate framework, plausible and mathematically operable in either a calculus- or algebra-based IPC. Within it, the sum of the velocity-dependent magnetic and the electric forces constitutes one observer invariant-electromagnetic force. It is important that this approach presents, in a certain sense, Galilean electromagnetism, as it is not built upon relativistic topics such as the invariance of the speed of light, simultaneity, kinematic and dynamic effects of Special Relativity. The treatment is entirely focused on electromagnetic phenomena. It conforms to the intuitive idea of force invariance, which makes it appropriate for an introductory and conceptually orientated course. The field transformations emerge from a need to account for simple physical situations, through bridging between points of view of observers in relative motion.

Almost 100 years after the theory of electromagnetism was essentially reconstructed, it is unacceptable to retain and present the magnetic force in its velocity-dependent form and ignore the superficial contradiction with the principles taught to students in the mechanics unit of the same course. Since the concepts of magnetic force and magnetic interac-

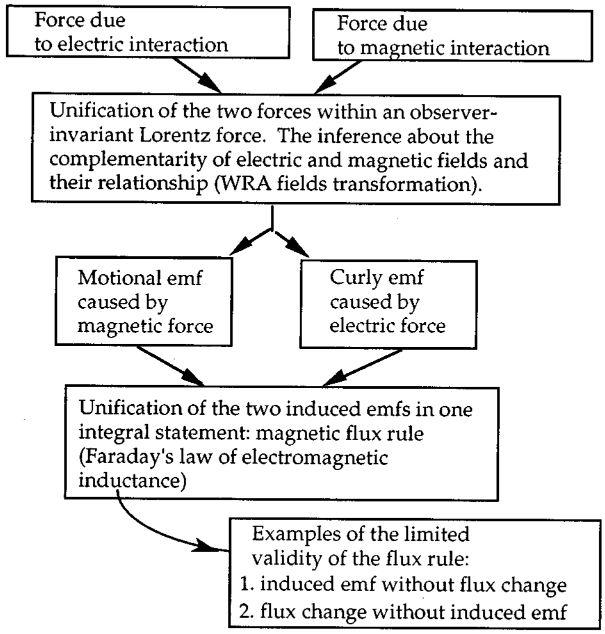

Fig. 12. A schematic representation of the suggested approach for teaching electromagnetism.

tion are inherently relativistic, a pedagogical problem arises: no consistent separation between ''classical'' (''compulsory'') and ''relativistic'' (''optional'') treatments of electromagnetic phenomena is possible. In this respect, electromagnetism is in principle different from gravitation where one can remove relativistic effects into a separate instructional unit. The reason for this difference is that two opposing electric charges cancel out the electrostatic effects which would otherwise prevail. Against the background of this cancellation, the effects of relativity become easily observable, common and ''nonrelativistic,'' as in the magnetic interaction. This situation is unique. A velocity-dependent term also appears within the relativistic theory of gravitational interaction.[66] The hypothesized discovery of an opposite gravitational charge could cause a parallel cancellation of the ''gravistatic'' interaction. In that case, the ''gravidynamic'' force could become observable at small velocities. We believe that it is not just physics majors who could comprehend this fact.

The presentation of the electromagnetic induction could be rearranged appropriately. The relative movement of a magnet and a conductor could be shown to cause two complementary phenomena depending on which observer is chosen. In simple cases, this choice determines whether the induced emf is caused by the magnetic or electric force (field). The complimentary nature of the two forces could be interpreted as reflecting the common nature of electric and magnetic fields. Following this route, the integral law of induction (the flux rule) is introduced as a reduction of two observed phenomena into one rule which is of less general validity due to its integral nature.[67] In introductory courses, the reduction to the flux rule can be made qualitatively, using simple cases (moving conducting rod).

Figure 12 presents a suggested sequence in teaching electromagnetism. We leave more elaboration of the suggested approach for further discussions.

We expect this approach to be efficient in promoting students' conceptual change from spontaneous views of separate magnetic and electric phenomena into a consistent synthetic view, in accordance with the present scientific knowledge of electromagnetism.

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[1] We draw our conclusions from a survey of more than ten currently employed IPC textbooks published in the USA, most of them recently.
[2] The Lorentz force is not unique in its velocity dependence. The Biot–Savart law is another example. We focused on the velocity dependence of the magnetic force as in many texts it is considered earlier than the sources of magnetic field. Force serves as a starting point for the suggested by us pedagogical changes. In their textbook *Electric and Magnetic Interactions* (Wiley, New York, 1995), R. Chabay and B. Sherwood start from the magnetic field and the Biot–Savart law, and from there develop the critique of velocity dependence (p. 459).
[3] L. C. McDermott, ''Millikan Lecture 1990: What we teach and what is learned—Closing the gap,'' Am. J. Phys. **59**, 301–315 (1991).
[4] For example, S. Rainson, G. Transtromer, and L. Viennot, ''Students' understanding of superposition of electric fields,'' Am. J. Phys. **62**, 1026–1032 (1994).
[5] I. Galili, ''Mechanics background for students' misconceptions in electromagnetism,'' Int. J. Sci. Ed. **17**, 371–387 (1995).
[6] A. Arons, *A Guide to Introductory Physics Teaching* (Wiley, New York, 1990), p. 191.
[7] Reference 4.
[8] A. K. T. Assis and F. M. Peixoto, ''On the Velocity in the Lorentz Force,'' The Phys. Teach. **30**, 480 (1992).
[9] M. Born, *Einstein's Theory of Relativity* (Dover, New York, 1965).
[10] For example, H. Benson, *University Physics* (Wiley, New York, 1991), p. 570; H. D. Young, *University Physics* (Addison-Wesley, Reading, MA, 1992), p. 773; R. A. Serway, *Physics* (Saunders College Publishing, Philadelphia, 1990), p. 806; P. Tipler, *Physics for Scientists and Engineers* (Worth, New York, 1990), p. 783; D. Halliday, R. Resnick, and J. Walker, *Fundamentals of Physics* (Wiley, New York, 1993), p. 819.
[11] R. Resnick, D. Halliday, and K. Krane, *Physics* (Wiley, New York, 1992), pp. 773–774.
[12] R. Chabay and B. Sherwood, in Ref. 2.
[13] E. M. Purcell, *Electricity and Magnetism* (McGraw-Hill New York, 1985), p. 208.
[14] Reference 8.
[15] D. Halliday, R. Resnick and J. Walker, in Ref. 10, p. 825.
[16] M. Alonso and E. J. Finn, *Physics* (Addison-Wesley, Reading, MA, 1992), p. 584.
[17] G. I. Cohn, ''Electromagnetic Induction,'' Electr.-Eng. **68**, 441–447 (May 1949). (Thanks to B. Sherwood).
[18] R. Feynman, R. B. Leighton, and M. Sands, *Feynman Lectures on Physics* (Addison–Wesley, Reading, MA, 1964), Vol. II.
[19] M. T. H. Chi, J. D. Slotta, and N. De Leeuw, ''From things to process: A theory of conceptual change for learning science concepts,'' Learn. Instrum. **4**, 27–43 (1994).
[20] O. Jefimenko, ''Force exerted on a stationary charge by a moving electric current or by a moving magnet,'' Am. J. Phys. **61**, 218–222 (1993).
[21] C. Kittel, W. D. Knight, and M. A. Ruderman, *Mechanics* (McGraw-Hill, New York, 1973), p. 368.
[22] J. F. Reichert, *A Modern Introduction to Mechanics* (Prentice Hall, Englewood Cliffs, NJ, 1991).
[23] Reference 13.
[24] We do not discuss, the consistency of this low velocity limit which would by far expand the scope and goals of this paper. After the second term in Eq. (8b) is neglected, the problem is fixed. [M. Bellac and I.-M. Levy-Leblond, ''Galilean Electromagnetism,'' Nuovo Cimento **14B**, 217–234 (1973).]
[25] J. Piaget, *Biology and Knowledge* (The University of Chicago Press, Chicago, 1974), p. 148.
[26] R. Chabay and B. Sherwood, in Ref. 2, pp. 482, 490–494.
[27] E. M. Purcell, in Ref. 13, Chaps. 5, 6; A. P. French, *Special Relativity* (Norton, New York), Chap. 8.
[28] For example, M. Alonso and E. J. Finn, in Ref. 16, p. 588.
[29] For example, M. Alonso and E. J. Finn, *Fundamental University Physics* (Addison-Wesley, Reading, MA, 1967), Vol. 2, p. 539.
[30] For example, P. Tipler, in Ref. 10, p. 813.
[31] R. Chabay and B. Sherwood, in Ref. 2.
[32] If one considered the two charges moving perpendicularly to each other,

the flaw in Newton's third law would become obvious.

[33]For example, H. Benson, in Ref. 10, p. 596.

[34]For example, P. Tipler, in Ref. 10, p. 812; R. Chabay and B. Sherwood, in Ref. 2, p. 436.

[35]Neglecting this term deserves a comment. Within our approximation, both terms in Eq. (8a) are of the same order. Hence, the term $1/c^2[\mathbf{v}\times\mathbf{E}]$ is of the order v^2/c^2.

[36]A. Arons, in Ref. 6.

[37]Dissatisfaction with prior knowledge, and the recognition of the new knowledge to be intelligible, plausible and fruitful, were claimed to be necessary conditions for a student to genuinely adopt the new knowledge in the progress of learning. See, for example, G. J. Posner, K. A. Strike, P. W. Hewson, and W. A. Gertzog, Sci. Educ. **66**, 211–227 (1982).

[38]For example, E. M. Purcell, in Ref. 13 or J. D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1965).

[39]This question, which naturally appears, is rarely addressed in IPC texts. The text of R. Resnick, D. Halliday, and K. Krane, in Ref. 11, pp. 793–795, is an exception.

[40]M. Born, in Ref. 9.

[41]M. N. Wise, ''The mutual embrace of electricity and magnetism,'' Science **203**, 1310–1318 (1979).

[42]A. I. Miller, ''Unipolar induction: A case study of the interaction between science and technology,'' Ann. Sci. **38**, 155–189 (1981).

[43]Reference 17.

[44]J. P. G. Hewitt, *Conceptual Physics* (Addison-Wesley, Reading, MA, 1982), p. 566.

[45]A. Einstein, ''On the Electrodynamics of the Moving bodies,'' Ann. Phys. **17**, 891 (1905). English translation in *The Principle of Relativity* (Dover, New York, 1952).

[46]See for example, M. Faraday, *Experimental Researches in Electricity*, in Great Books of the Western World Vol. 45 (Encyclopedia Britannica, Chicago, 1978), Sec. 6.

[47]E. M. Purcell, in Ref. 13, Chap. 7.

[48]The customary integral form of the law of inductance was written first by F. Neumann (1798–1895) in 1845.

[49]M. Born, in Ref. 9; A. Einstein in Ref. 45.

[50]Of course, one can still correctly talk about ''moving field'' regarding electromagnetic-waves radiation by accelerated charges.

[51]One can easily find such definitions as ''the induced emf equals the number of field lines cut per second by a conductor.''

[52]The texts by E. M. Purcell, in Ref. 13 and R. Feynman, R. B. Leighton, and M. Sands, in Ref. 18, are not IPC textbooks and do not address a novice learner.

[53]J. D. Jackson, in Ref. 38, p. 173.

[54]E. M. Purcell, in Ref. 13, Chap. 7, p. 243.

[55]P. Lorrain and D. Corson, *Electromagnetic Fields and Waves* (Freeman, New York, 1970), p. 341, Eq. (8–36).

[56]Reference 17.

[57]For example, F. W. Sears, M. W. Zemansky, and H. D. Young, *University Physics* (Addison-Wesley, Reading, 1982), p. 630; H. C. Ohanian, *Physics* (Norton, New York, 1989), p. 780.

[58]R. Chabay and B. Sherwood, in Ref. 2, Chap. 13.

[59]They constitute a great majority. See, for example, R. Resnick, D. Halliday, and K. Krane, in Ref. 11; P. Tipler, in Ref. 10; D. C. Giancoli, *Physics for Scientists and Engineers* (Prentice Hall, Englewood Cliffs, NJ, 1989); A. Hudson and R. Nelson, *University Physics* (Saunders College Publishing, Philadelphia, 1990).

[60]Faraday's disc should not be confused with the case of unipolar induction. In the latter the rotating disc is a magnet itself. This case is much more complicated conceptually and never touched on in introductory physics courses, e.g., L. D. Landau and E. M. Lifshits, *Theoretical Physics*, Vol. VIII, ''The Electrodynamics of Continuous Media'', Oxford: Pergamon Press, 1963, p. 209.

[61]A straightforward integration of the magnetic force felt by the electrons along the radius of the disc leads to the well-known result for the induced emf:emf$=\frac{1}{2}\omega BR^2$.

[62]For example, R. Resnick, D. Halliday, and K. Krane, in Ref. 11; R. Wolfson and J. M. Pasachoff, *Physics for Scientists and Engineers* (Harper Collins College Publishers, New York, 1995).

[63]H. Benson, p. 628; H. D. Young, p. 845, both in Ref. 10.

[64]R. Feynman, R. B. Leighton, and M. Sands, in Ref. 18, pp. 17–2, 3.

[65]Studies of teaching electricity preceded the suggestion of a new conceptual approach to learning electrical circuits. See, for example, H. Haertel, Report No. IRL 87-0001, Palo Alto: Institute on Research on Learning (1987) and the recently published textbook by R. Chabay and B. Sherwood, in Ref. 2.

[66]L. D. Landau and E. M. Lifshiz, *The Classical Theory of Fields* (Addison-Wesley, Reading, 1959), p. 279.

[67]As was strongly stressed by Feynman (Ref. 18).

IF IT'S NONLINEAR, PUNT

That gives me three equations and five unknowns.

Now all I have to do is show how *x* varies with time, so I can graph the position of the balloon and Greene will think I'm a hero.

Step 6. Dust off differential equations books from sophomore year at Hopkins. A differential equation deals with things that move in time, like the piston of an engine, the earth, and the moon—or the skin of a balloon being blown up.

Everything in the differential equations book deals with ''linear'' differential equations. That means that the second term, the *x*-dot-squared term, means I can't solve this using any methods in that book. I'm multiplying the velocity by the velocity, and that makes it ''nonlinear.''

Step 7: Punt. This is what you do at MIT when the institute or the problem set has painted you into a corner.

Pepper White, *The Idea Factory—Learning to Think at MIT* (Penguin Books, New York, 1991), pp. 104–105.